INTRODUCTION TO MACHINE DESIGN

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

CLASSIFICATIONS OF MACHINE DESIGN

THE MACHINE DESIGN MAY BE CLASSIFIED AS FOLLOWS:

1. ***Adaptive design***. In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design***. This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design***. This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(a) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(*e*) ***System design.*** It is the design of any complex mechanical system like a motor car.

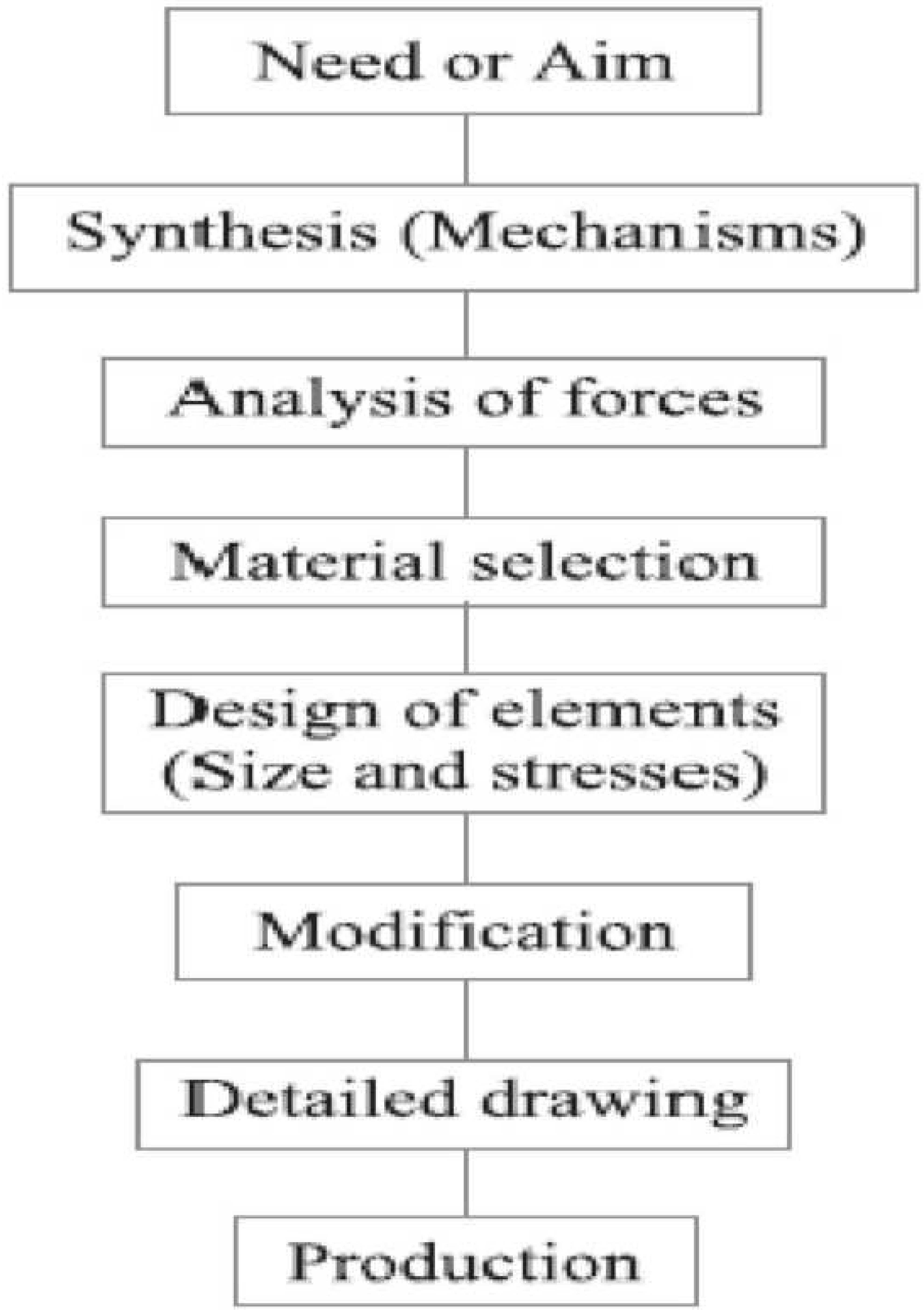
(*f*) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

GENERAL CONSIDERATIONS IN MACHINE DESIGN

1. *Type of load and stresses caused by the load.*
2. *Motion of the parts or kinematics of the machine.*
3. *Selection of materials.*
4. *Form and size of the parts.*
5. *Frictional resistance and lubrication.*
6. *Convenient and economical features.*
7. *Use of standard parts.*
8. *Safety of operation.*
9. *Workshop facilities.*
10. *Number of machines to be manufactured.*
11. *Cost of construction.*
12. *Assembling.*

GENERAL PROCEDURE IN MACHINE DESIGN



Steps

1. ***Recognition of need***. First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis (Mechanisms)***. Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces***. Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection***. Select the material best suited for each member of the machine.

5. ***Design of elements (Size and Stresses)*** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member of the machine limit.

6. ***Modification***. Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. ***Detailed drawing***. Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. ***Production***. The component, as per the drawing, is manufactured in the workshop.

ENGINEERING MATERIALS AND THEIR PROPERTIES

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

CLASSIFICATION OF ENGINEERING MATERIALS

- The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

- The metals may be further classified as:

(a) Ferrous metals and (b) Non-ferrous metals.

- The **ferrous metals are those which have the iron as their main constituent, such as cast iron ,* wrought iron and steel.
- The *non-ferrous metals are those which have a metal other than iron as their main* constituent, such as copper, aluminum, brass, tin, zinc, etc.

SELECTION OF MATERIALS FOR ENGINEERING PURPOSES

- The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

- The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

PHYSICAL PROPERTIES OF METALS

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point.

MECHANICAL PROPERTIES OF METALS

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. **Strength.** It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is
called stress.

2. **Stiffness.** It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. **Elasticity.** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. **Plasticity.** It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. **Ductility.** It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as *fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test.

The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

WORKING STRESS, YIELD STRESS, ULTIMATE STRESS AND FACTOR OF SAFETY

1. **Ultimate stress.-**The stress, which attains its maximum value is known as ultimate stress. It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

2. **Yield stress.-**If the material is stressed beyond point the elastic limit, the plastic stage will reach i.e. on the removal of the load, the material will not be able to recover its original size and shape. Where the strain increases at a faster rate with any increase in the stress. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. The stress corresponding to yield point is known as yield point stress.

3. **Working Stress.** When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

Note : By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.

4. **Factor of Safety.** It is defined, in general, as the ratio of the maximum stress to the working stress.

Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

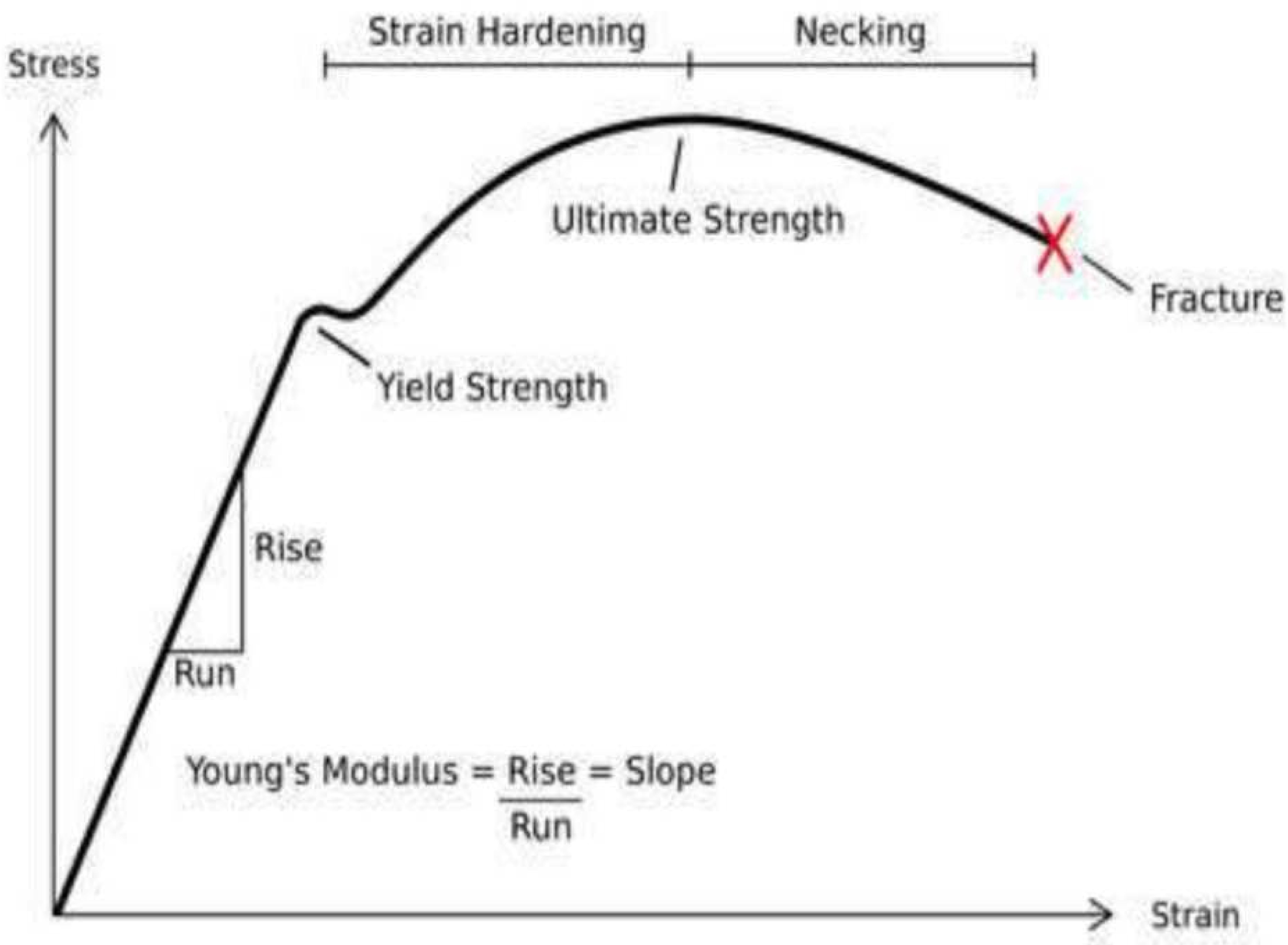
$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

Note: The above relations for factor of safety are for static loading.

STRESS STRAIN DIAGRAM:



Ductile Materials:

Ductile materials are those which are capable of having large strains before they are fractured. Ductile materials can withstand high stress and are also capable of absorbing large amount of energy before their failure. A ductile material has a large Percentage of elongation before failure.

Some examples of ductile materials are aluminum, mild steel and some of its alloys i.e. copper, magnesium, brass, nickel, bronze and many others.

Different Points On Stress Strain Curve:

• Proportional Limit (σPL)

Proportional limit is the point on stress strain curve which shows the highest stress at which Stress and Strain are linearly proportional to each other where the proportionality constant is E known as modulus of elasticity. Above this point, stress is no longer linearly proportional to strain. On stress strain curve, proportional limit is shown by P. It is denoted by σPL.

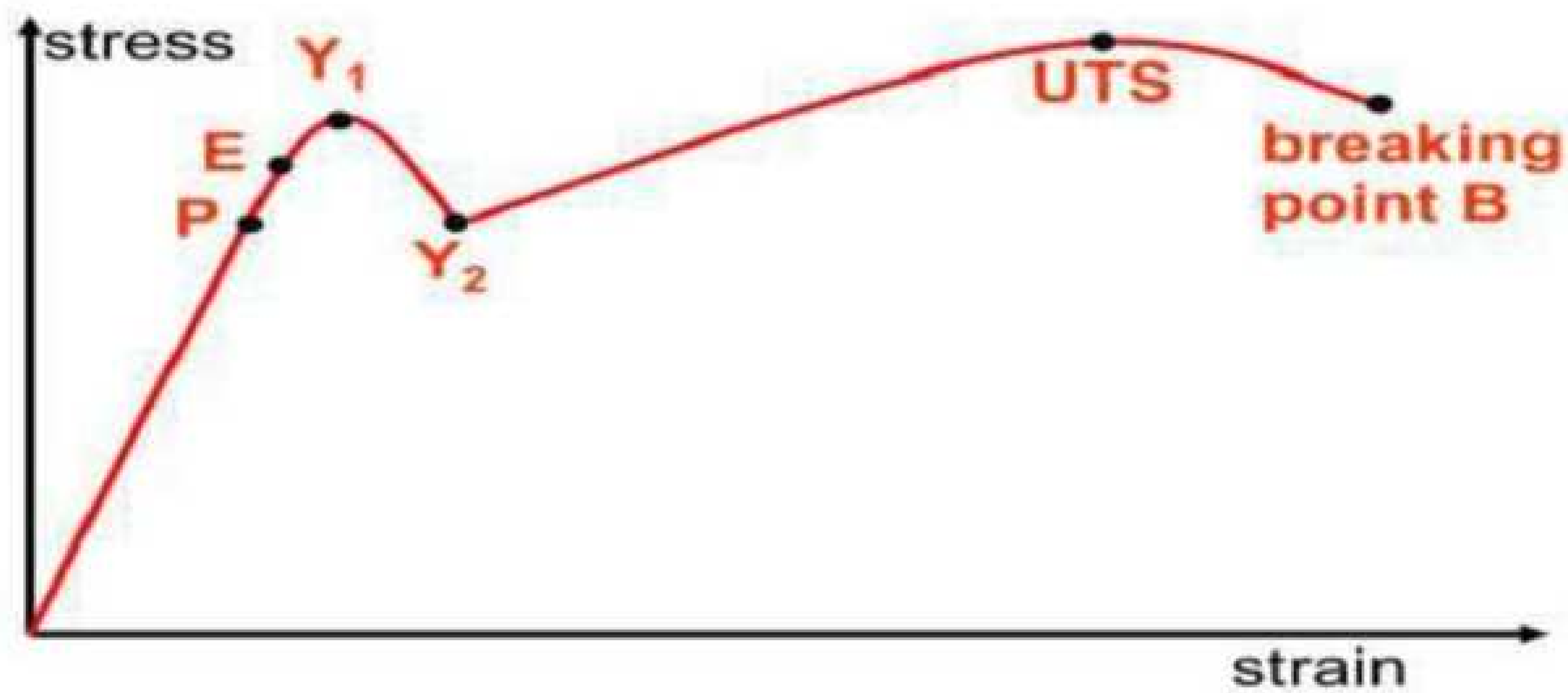
For annealed mild steel the limit of proportionality occurs at 230 MPa. The graph shows that the length of graph up to proportional limit (P) is a straight line which means that up to proportional limit stress is linearly proportional to strain.

• Elastic Limit (σEL)

Elastic limit is the point which shows the maximum stress that can be applied to the body without resulting in permanent deformation when stress is removed. At elastic limit when the load is removed from the body, it returns to original size and shape. At elastic limit stress is no longer linearly proportional to strain. It is denoted by σEL. For stress strain graph of mild steel, elastic limit is just close to proportional limit.

• Yield point (σY)

Yield point is the point which shows the stress at which a little or no increase in stress results to large increase in strain that is material continues to deform without increase in load. At this point the material will have permanent deformation. It is denoted by σY. For steel, yield point is also just above proportional limit.



Yield point is of two types:

- o Upper yield point.
- o Lower yield point.

Upper yield point is shown by Y1 and lower yield point is shown by Y2 as in diagram given above:

Among the common materials, only steel exhibits yield point. For annealed mild steel, upper yield point occurs at 260 MPa and lower yield point occurs at 230 MPa.

• **Ultimate Tensile Strength (σU)**

As the stress on material is increased further, the stress and the strain increases from yield point to a point called ultimate tensile strength (UTS) where stress applied is maximum. Thus ultimate tensile strength can be defined as the highest stress on the specimen which it can withstand. For annealed mild steel, ultimate tensile strength occurs at 400 MPa. It is denoted by σU.

• **Fracture Stress (σF)**

After ultimate tensile strength, the applied stress decreases until the stress is obtained where material fractures called fracture stress. Fracture stress is also called breaking strength. It is denoted by σF.

WELDED JOINTS

Introduction

- A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

- Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium *e.g. to reunite metal at a crack, to build up* a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

ADVANTAGES AND DISADVANTAGES OF WELDED JOINTS OVER RIVETED JOINTS

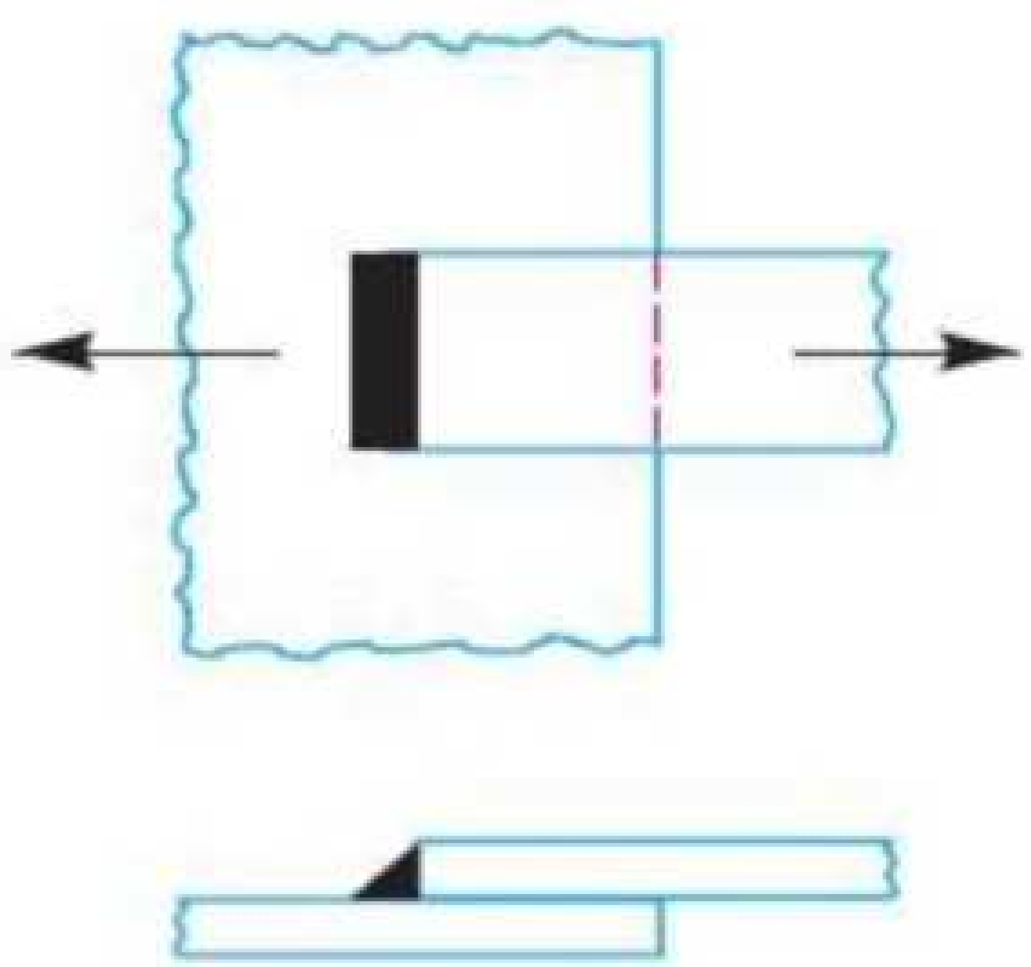
1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (*i.e. circular steel pipes) that they afford* difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

DISADVANTAGES AND DISADVANTAGES OF WELDED JOINTS OVER RIVETED JOINTS

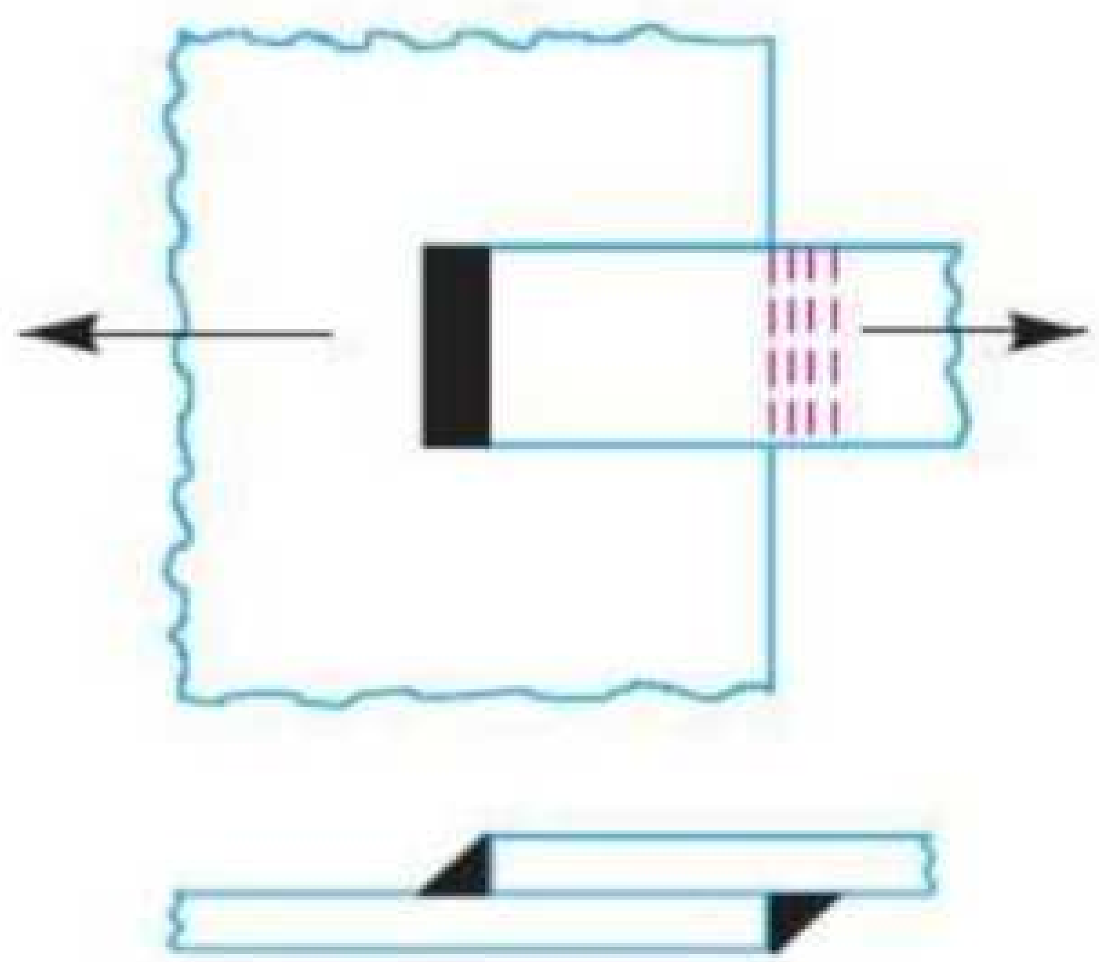
1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. The inspection of welding work is more difficult than riveting work.
4. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
5. The inspection of welding work is more difficult than riveting work.

TYPES OF WELDED JOINTS

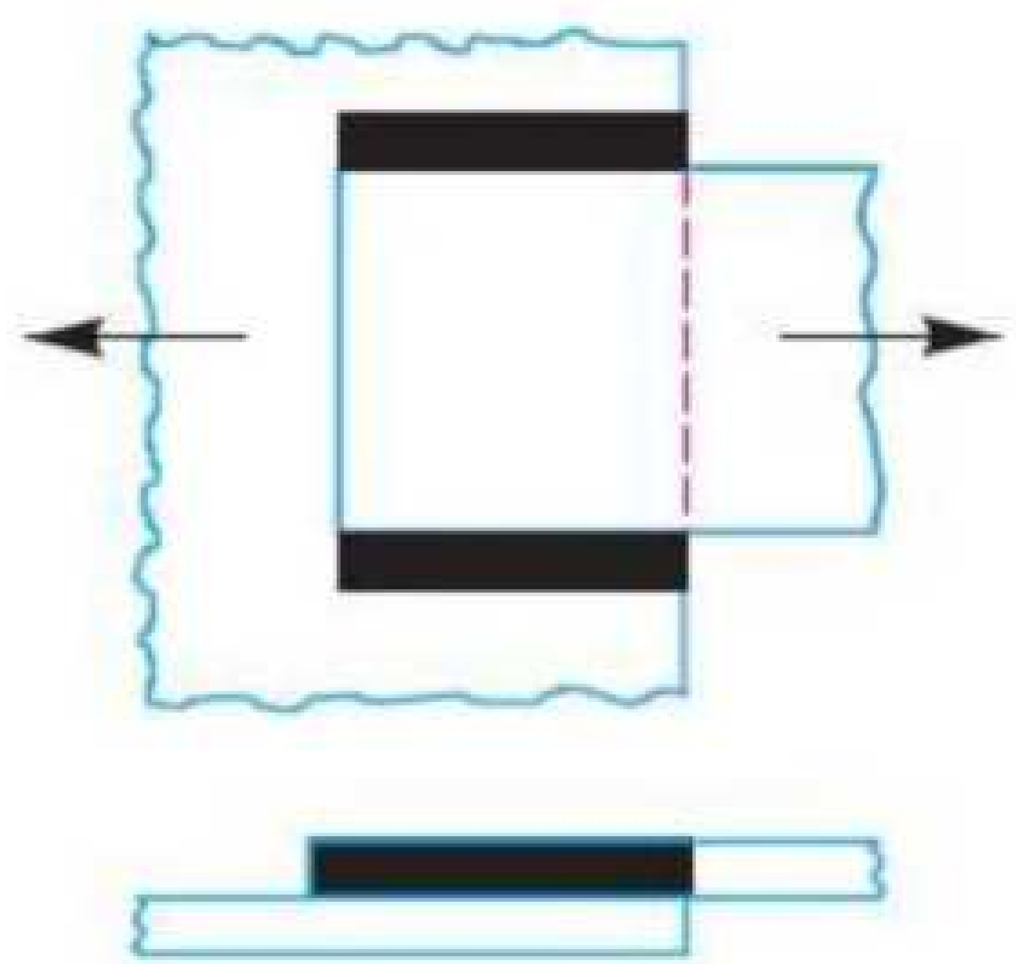
- The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be
- **1. Single transverse fillet,**
- **2. Double transverse fillet, and**
- **3. Parallel fillet joints.**



(*a*) Single transverse.



(*b*) Double transverse.

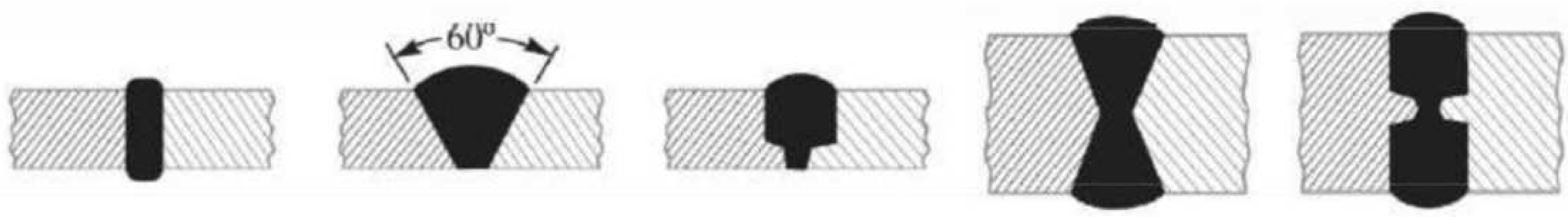


(*c*) Parallel fillet.

TYPES OF WELDED JOINTS

BUTT JOINT

- The butt joint is obtained by placing the plates edge to edge. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides. The butt joints maybe
- **1. Square butt joint**
- **2. Single V-butt joint**
- **3. Single U-butt joint**
- **4. Double V-butt joint**
- **5. Double U-butt joint**

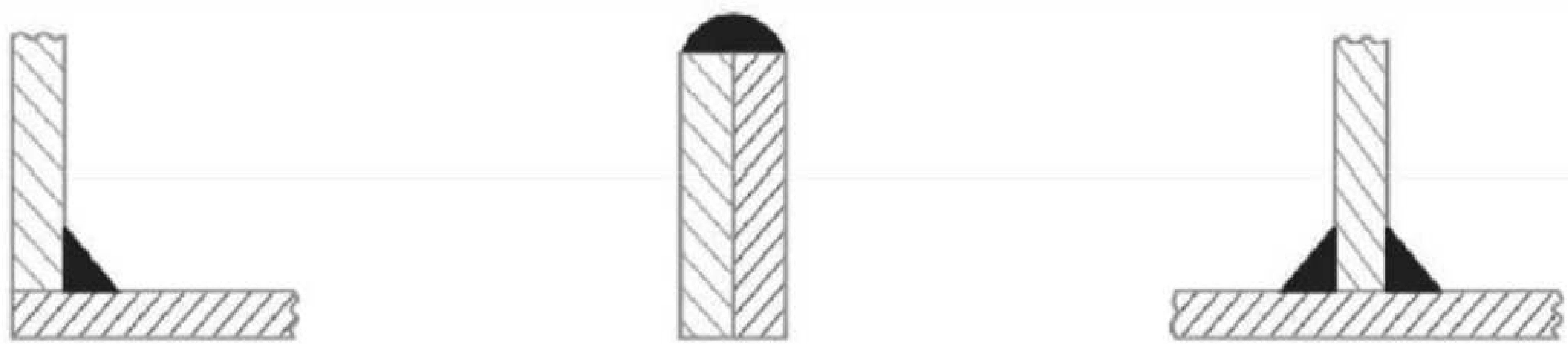


(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

TYPES OF WELDED JOINTS

Other Joints

- The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig. below.

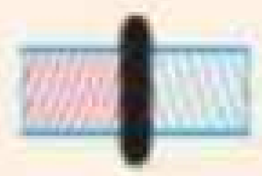
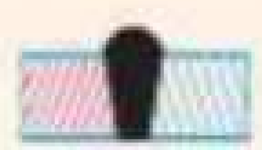
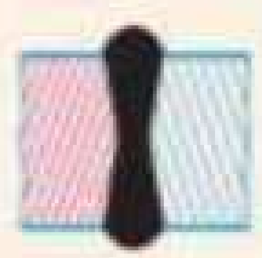
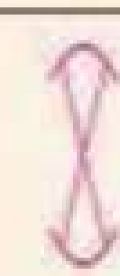
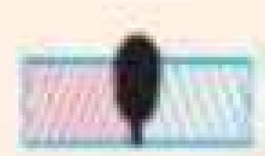
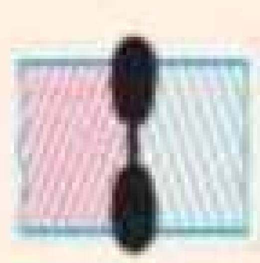
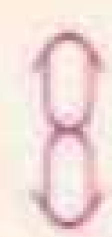
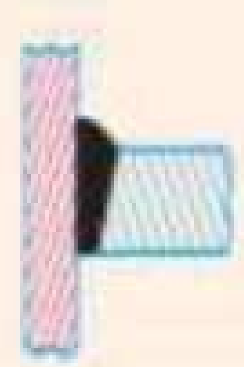
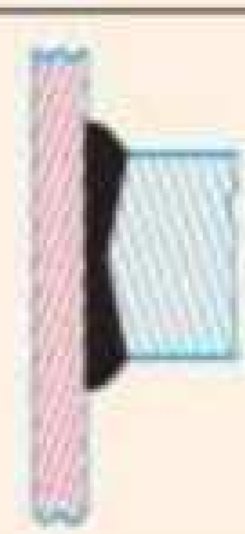


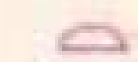
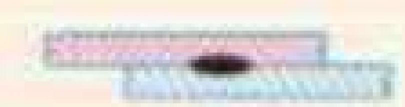
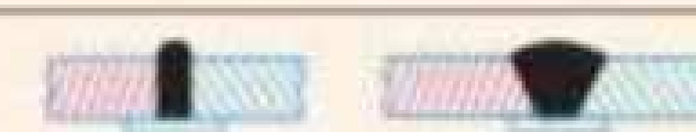
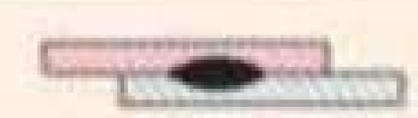
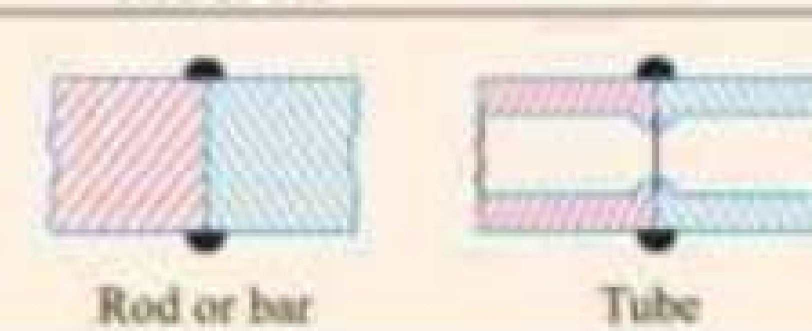
(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

The main considerations involved in the selection of weld type are:

1)The shape of the welded component required,
2)The thickness of the plates to be welded, and
3)The direction of the forces applied.

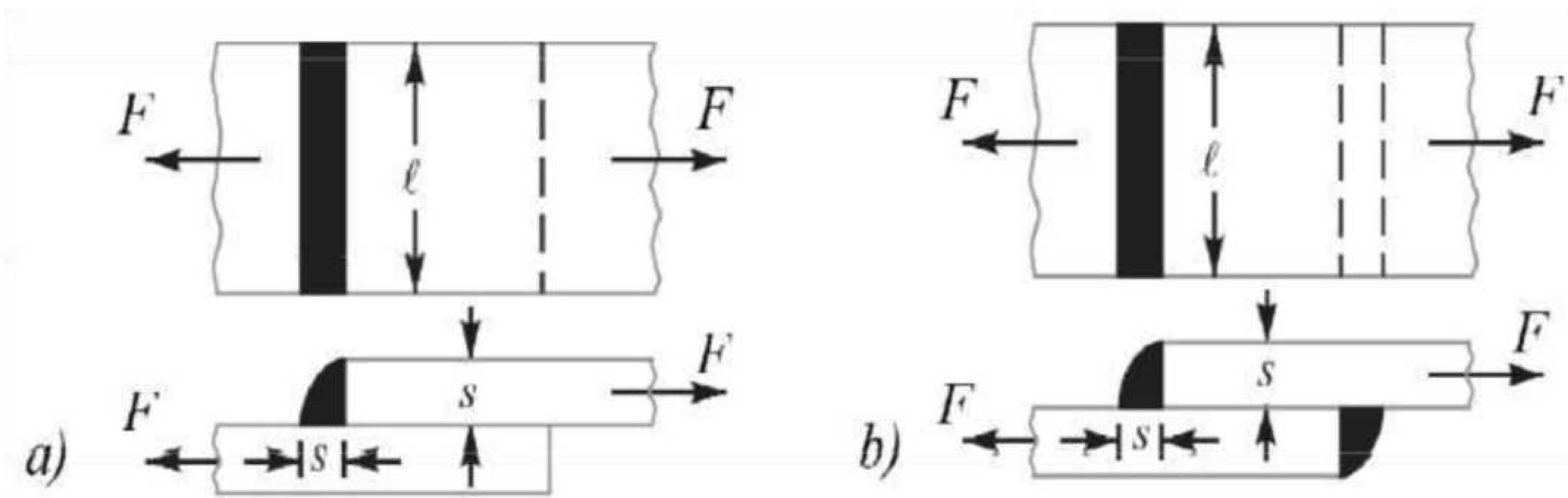
Basic Weld Symbols

S. No.	Form of weld	Sectional representation	Symbol
1.	Fillet		
2.	Square butt		
3.	Single-V butt		
4.	Double-V butt		
5.	Single-U butt		
6.	Double-U butt		
7.	Single bevel butt		
8.	Double bevel butt		

S. No.	Form of weld	Sectional representation	Symbol
9.	Single-J butt		
10.	Double-J butt		
11.	Bead (edge or seal)		
12.	Stud		⊥
13.	Sealing run		○
14.	Spot		Ж
15.	Seam		
16.	Mashed seam	 Before After	
17.	Plug		
18.	Backing strip		=
19.	Stitch		
20.	Projection	 Before After	△
21.	Flash	 Rod or bar Tube	И
22.	Butt resistance or pressure (upset)	 Rod or bar Tube	|

Strength of Transverse Fillet Welded Joints

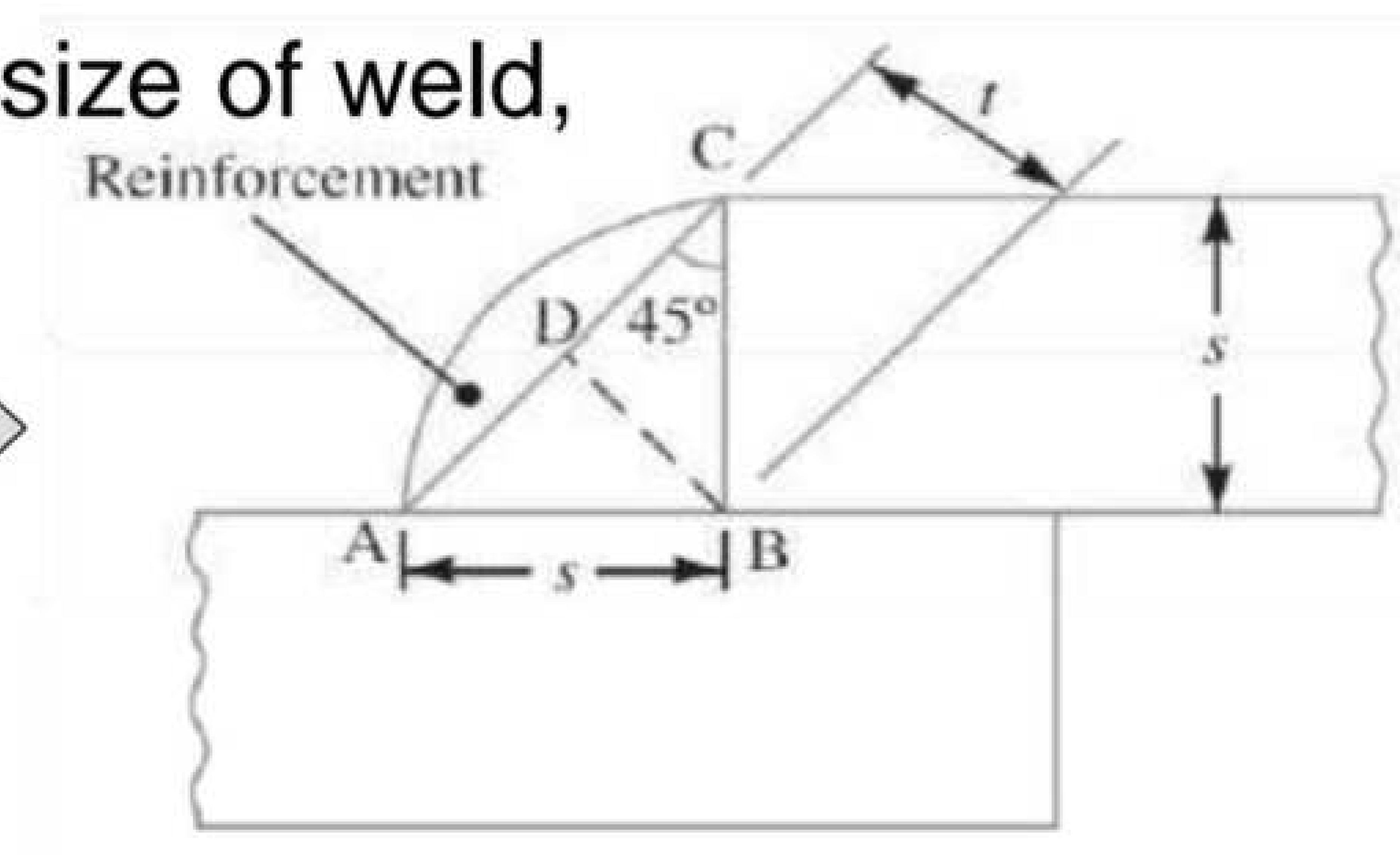
The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. (*a) and (b) respectively.*



In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC.

The enlarged view of the fillet is shown in Fig. below The length of each side is known as leg or size of the weld and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as throat thickness. The minimum area of the weld is obtained at the throat BD, which is given by the product of the throat thickness and length of weld.

t = Throat thickness (BD), s= Leg or size of weld,
= Thickness of plate, and
l= Length of weld,



From Fig. below, we find that the throat thickness,

Minimum area of the weld or throat area,

A = Throat thickness × Length of weld = $t \times l = 0.707\, s \times l$

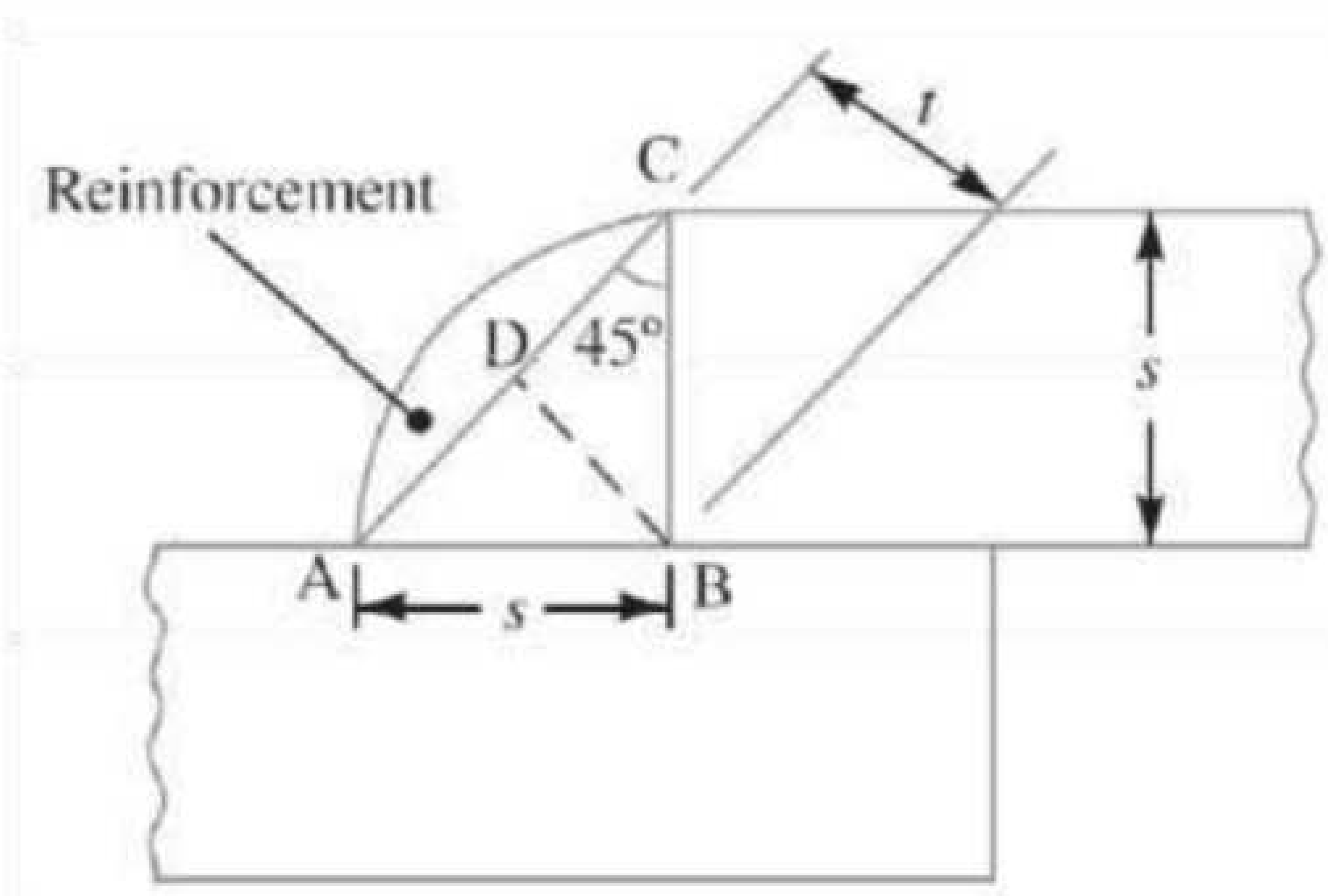
$$t = s \times \sin 45° = 0.707\, s$$

If σ_t is the allowable tensile stress for the weld metal, then the **tensile strength of the joint for single fillet weld,**

$$\mathbf{F = Throat\ area \times Allowable\ tensile\ stress = 0.707\, s \times l \times \sigma_t}$$

and **tensile strength of the joint for double fillet weld,**

$$\mathbf{F = 2 \times 0.707\, s \times l \times \sigma_t = 1.414\, s \times l \times \sigma_t}$$



Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig. (*a*). *The* minimum area of weld or the throat area,

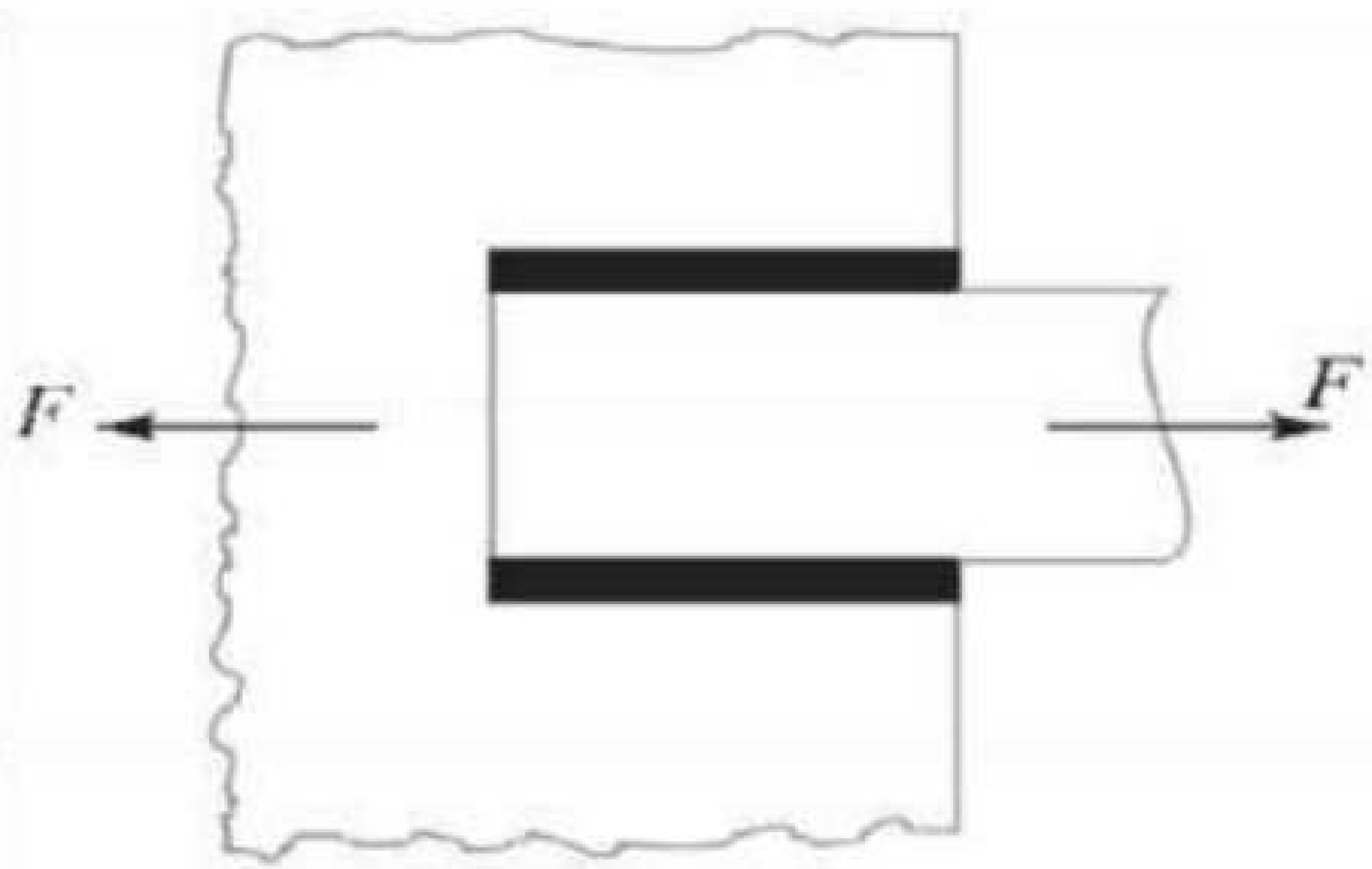
$$A = 0.707\ s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

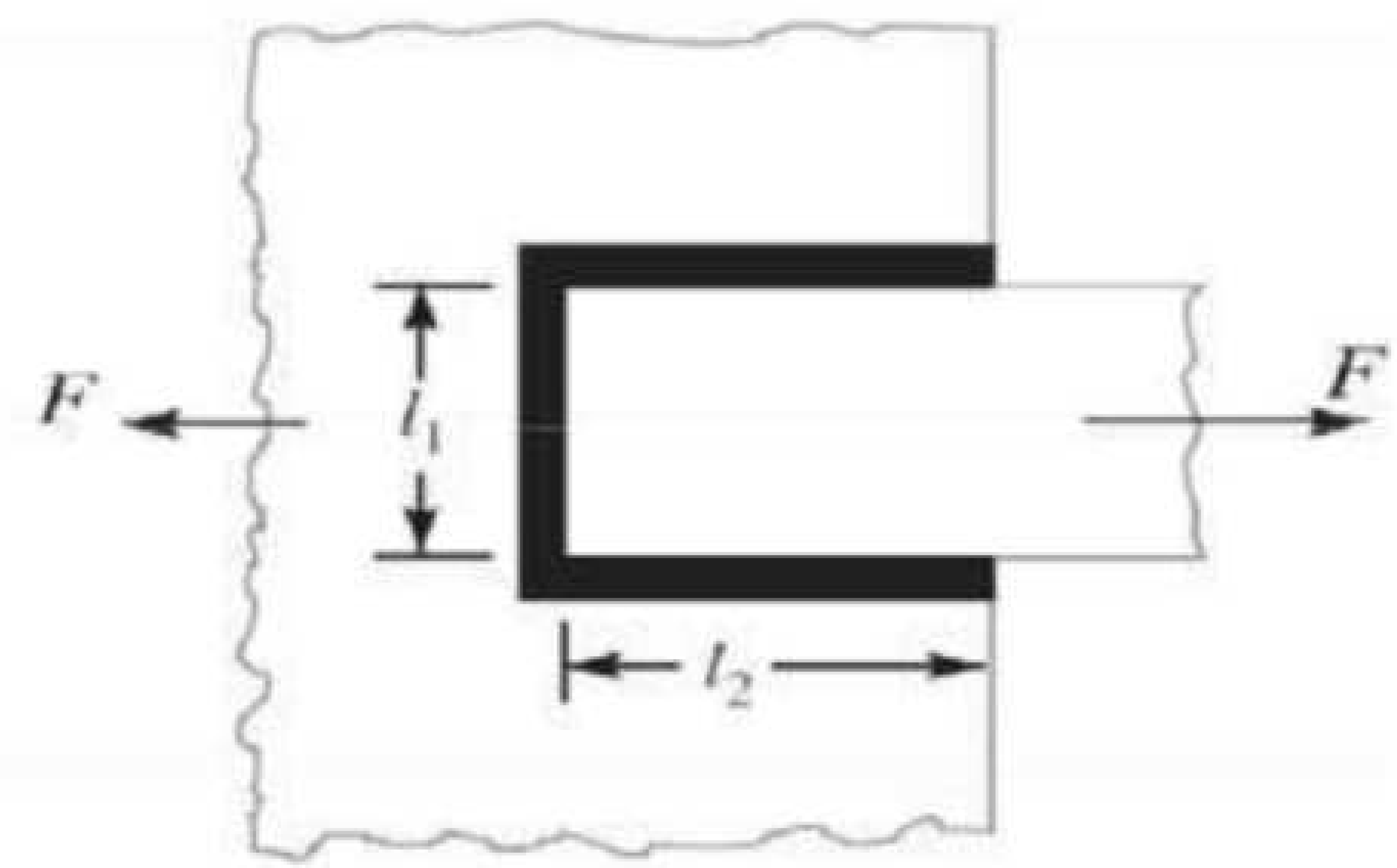
F = Throat area × Allowable shear stress = 0.707 s × l × τ

and shear strength of the joint for double parallel fillet weld,

F= 2 × 0.707 ×s × l × τ = 1.414 s × l × τ



(*a*) Double parallel fillet weld.



(*b*) Combination of transverse and parallel fillet weld.

Notes:

1. If there is a combination of single transverse and double parallel fillet welds as shown in Fig.,then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds.

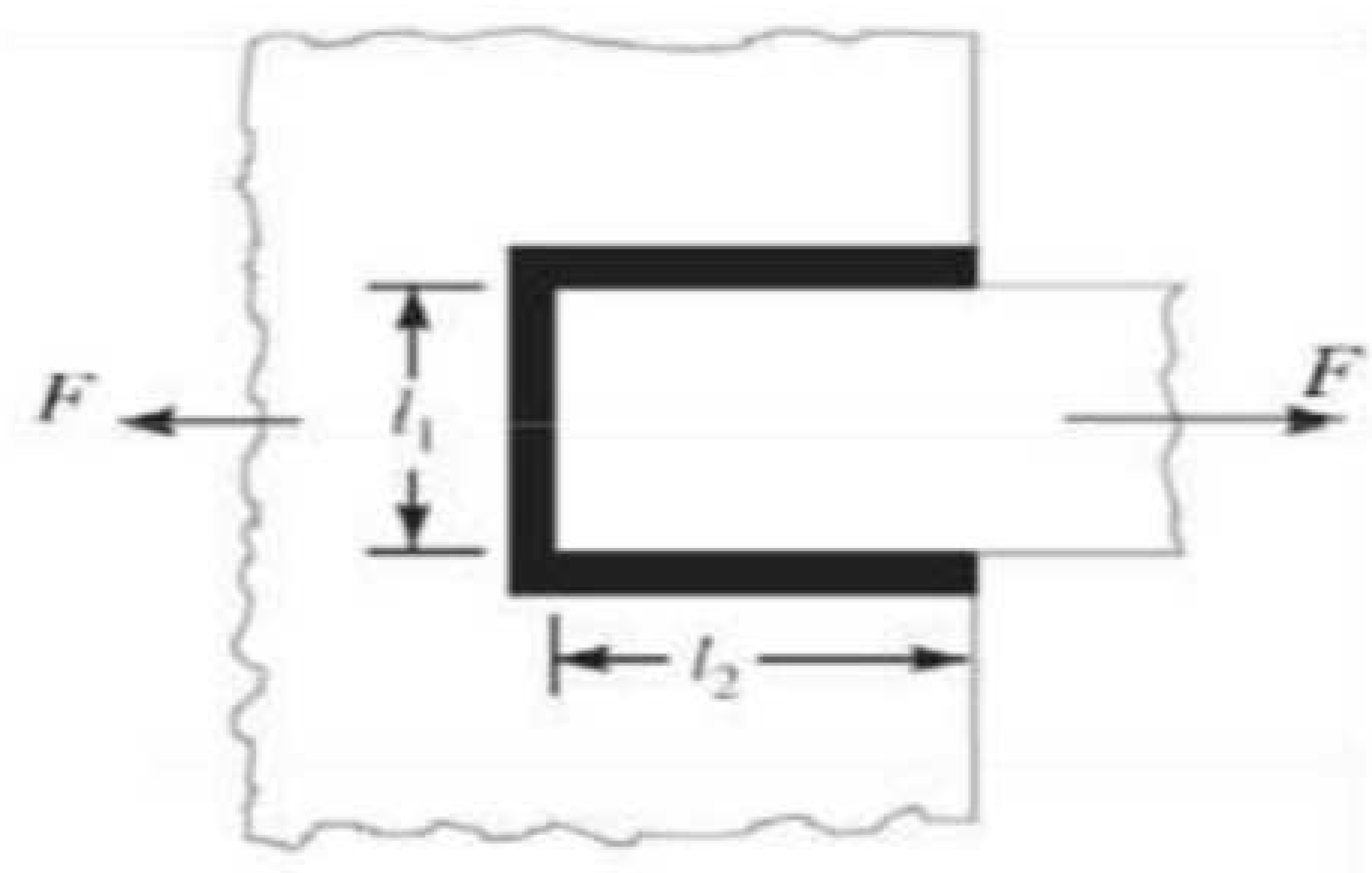
Mathematically,

$$P = 0.707s \times l1 \times \sigma t + 1.414\ s \times l2 \times \tau$$

where *l1 is normally the width of the plate.*

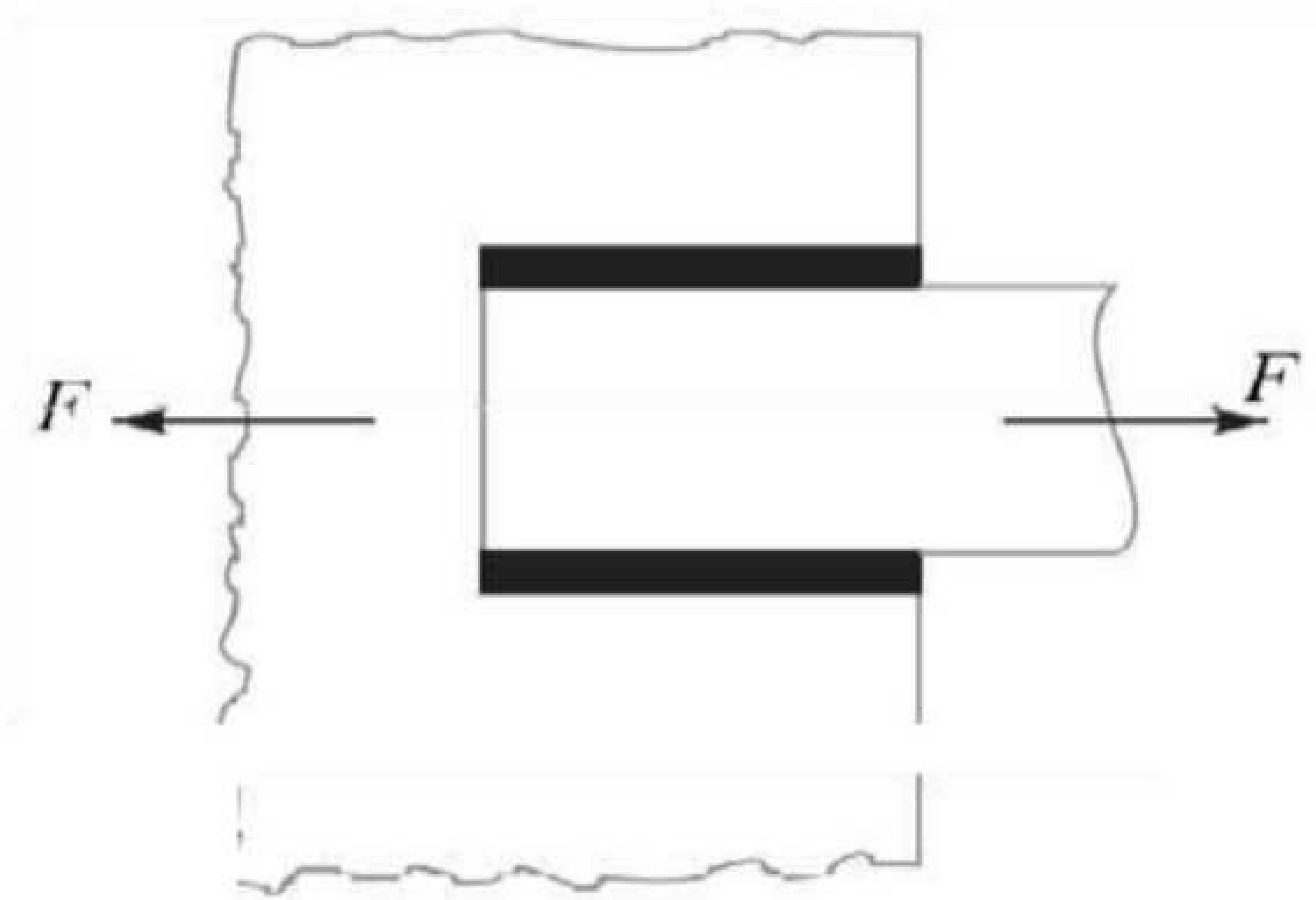
2. In order to allow for starting and stopping of the bead, **12.5 mm** should be added to the length of each weld obtained by the above expression.

3. For reinforced fillet welds, the throat dimension may be taken as 0.85 *t*.



(*b*) Combination of transverse and parallel fillet weld.

Q.1. A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.



Double parallel fillet weld.

ANSWER

Given- Width of the plate = 100 mm.

Thickness of the Plate = 10 mm.

P (Static Load) = 80 kN = 80 X 10^3 N

τ (Permissible Shear Stress) = 55MPa= 55 N/mm^2

Length of the Weld, L = ?

We have,

P= 2 × 0.707 ×s × l × τ = 1.414 s × l × τ

80 × 10^3 = 1.414 × *s* × *l* × *τ* = *1.414* × *10* × *l* × *55* = *778 l*

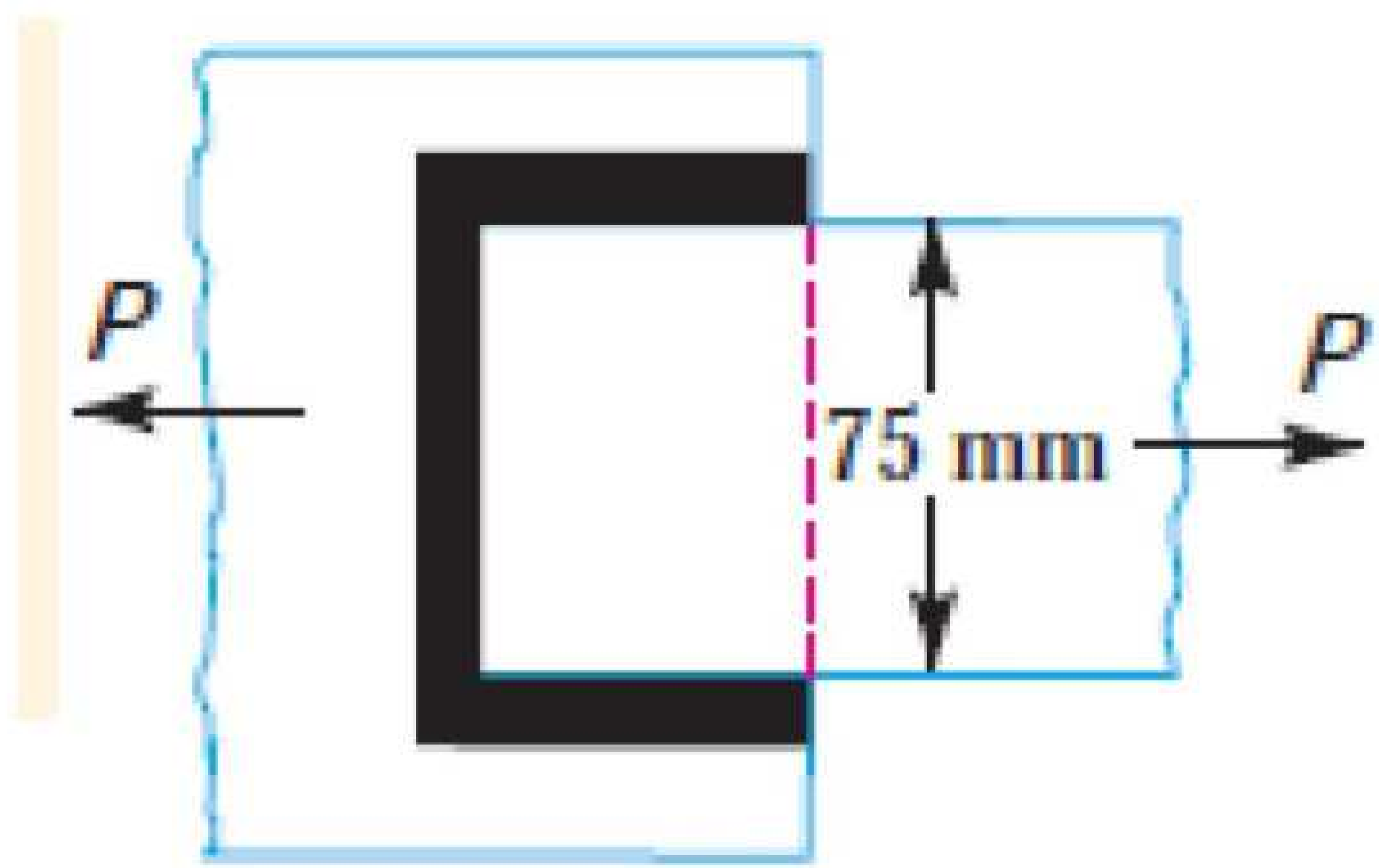
l = *80* × *103* / *778* = *103 mm*

Adding 12.5 mm for starting and stopping of weld run, we have

l = *103* + *12.5* = *115.5 mm* ***Ans.***

Q.2. ***A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to static loading.(Home Work)***

Hint- $P = 0.707s \times l1 \times \sigma t + 1.414\, s \times l2 \times \tau$



Eccentrically Loaded Welded Joints

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a joint, then maximum stresses are as follows:

Maximum normal stress,

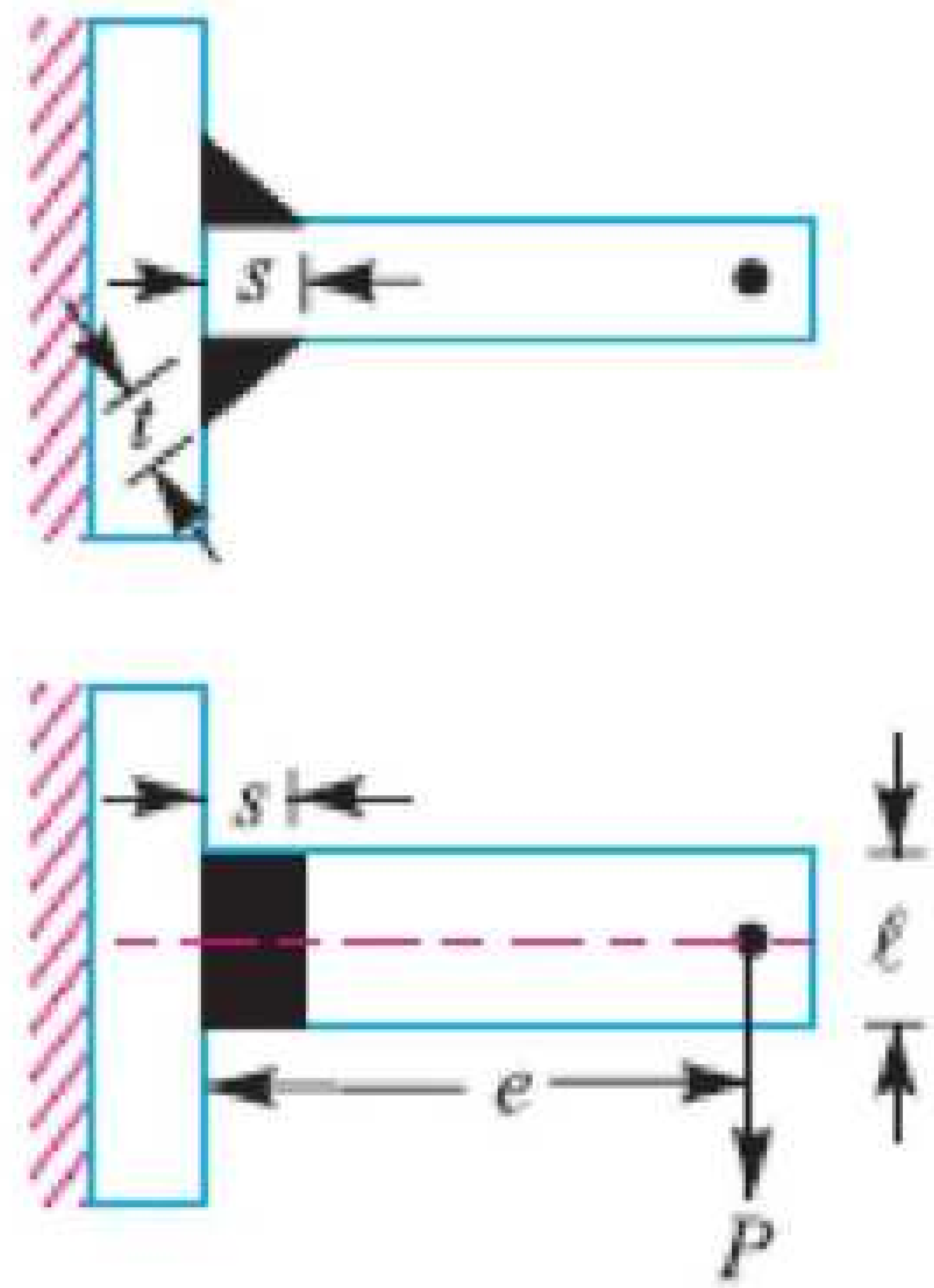
$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where σ_b = Bending stress, and

τ = Shear stress.



Consider a *T*-joint fixed at one end and subjected to an eccentric load *P* at a distance *e* as shown in Fig. 10.22.

Let s = Size of weld,

l = Length of weld, and

t = Throat thickness.

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force *P* acting at the welds, and
2. Bending stress due to the bending moment $P \times e$.

We know that area at the throat,

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l \times 2 = 2\,t \times l \qquad \text{... (For double fillet weld)}$$

$$= 2 \times 0.707\,s \times l = 1.414\,s \times l \qquad \text{... } (\because\ t = s \cos 45° = 0.707\,s)$$

∴ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\, s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \qquad \text{...(For both sides weld)}$$

$$= \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

∴ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\, P \times e}{s \times l^2}$

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

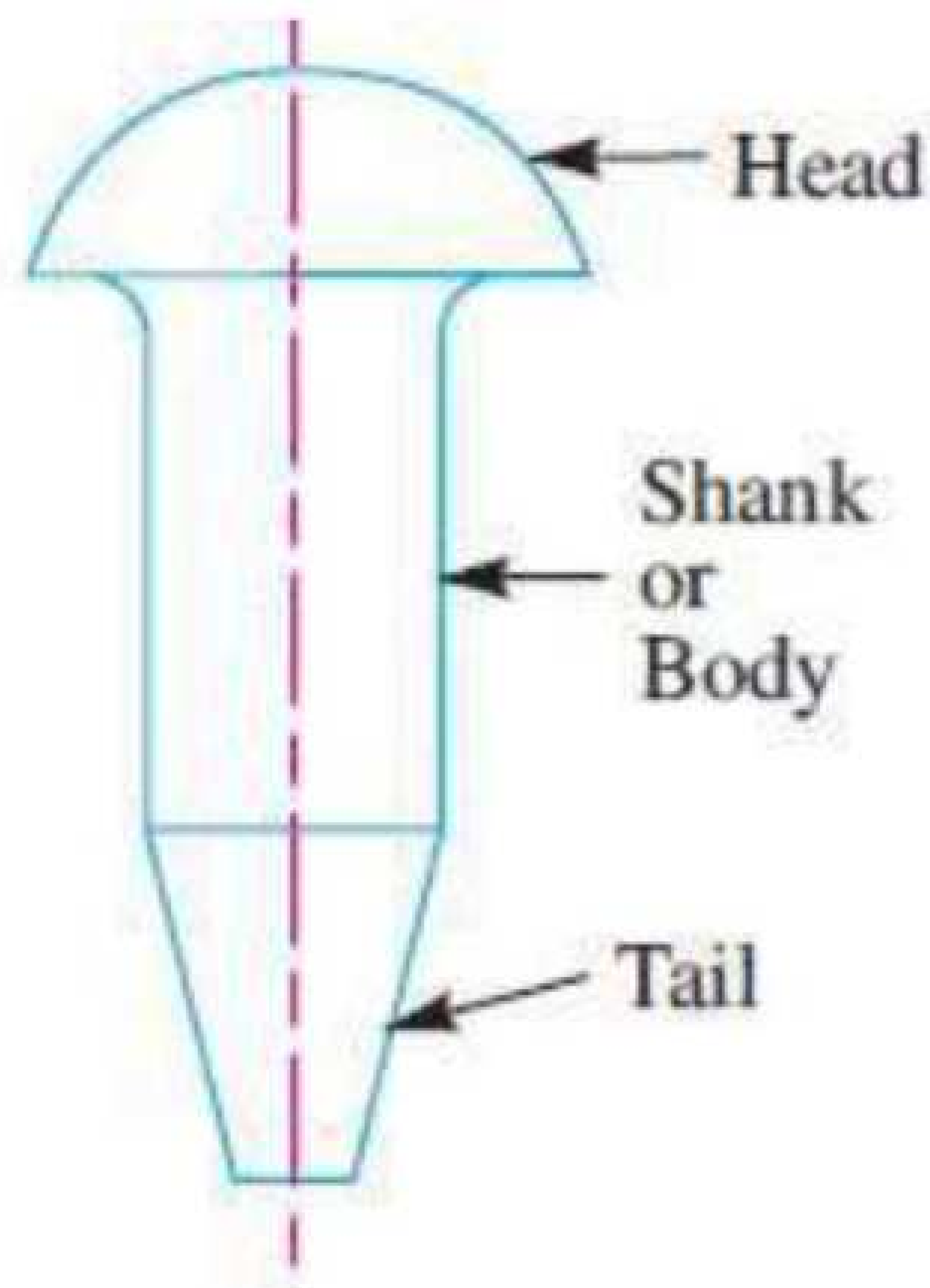
and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

RIVETS

INTRODUCTION

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called ***shank or body*** and lower portion of shank is known as ***tail***, ***as shown in Fig***. The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

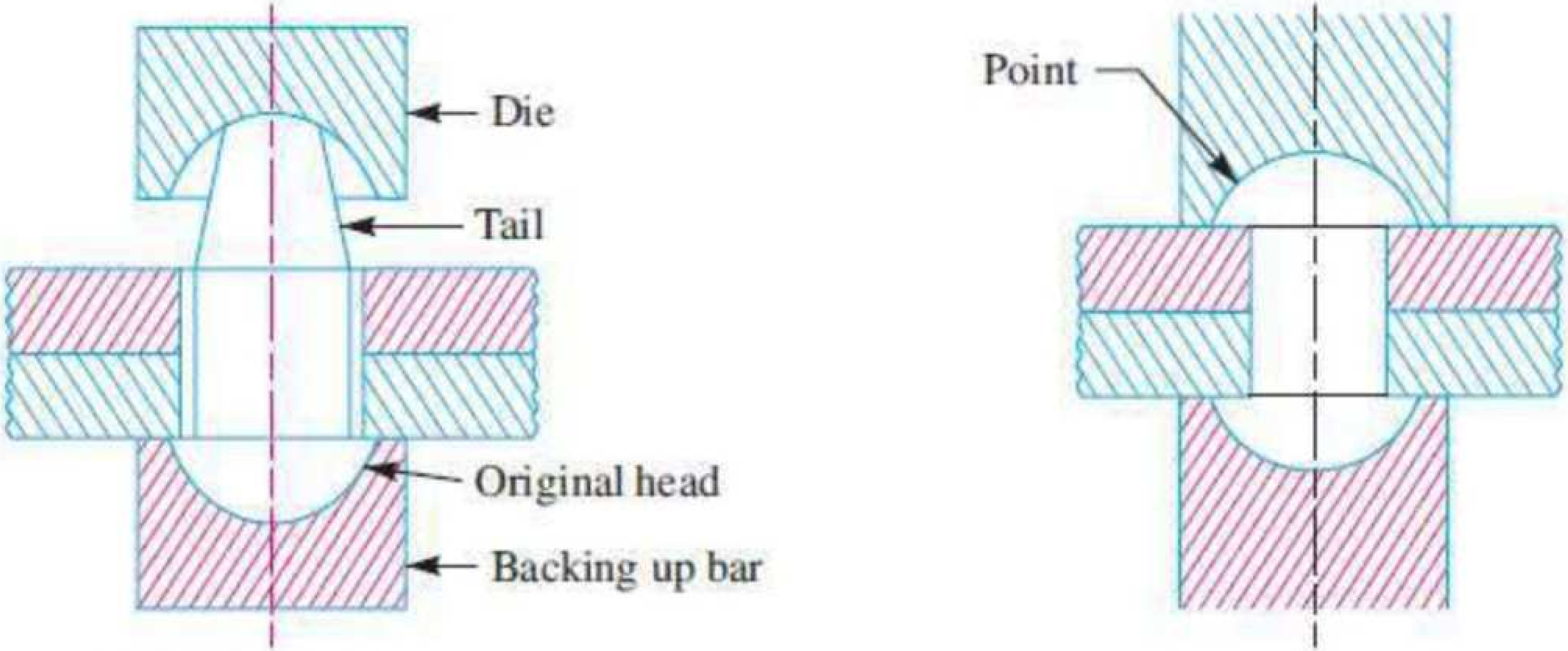


The fastenings (*i.e. joints)* may be classified into the following two groups :

1. Permanent fastenings, and
2. Temporary or detachable fastenings.

The ***permanent fastenings*** *are those fastenings which can not be disassembled without* destroying the connecting components. The examples of permanent fastenings in order of strength are soldered, brazed, welded and riveted joints.

The ***temporary or detachable fastenings*** *are those fastenings which can be disassembled* without destroying the connecting components. The examples of temporary fastenings are screwed, keys, cotters, pins and splined joints.



(a) Initial position.

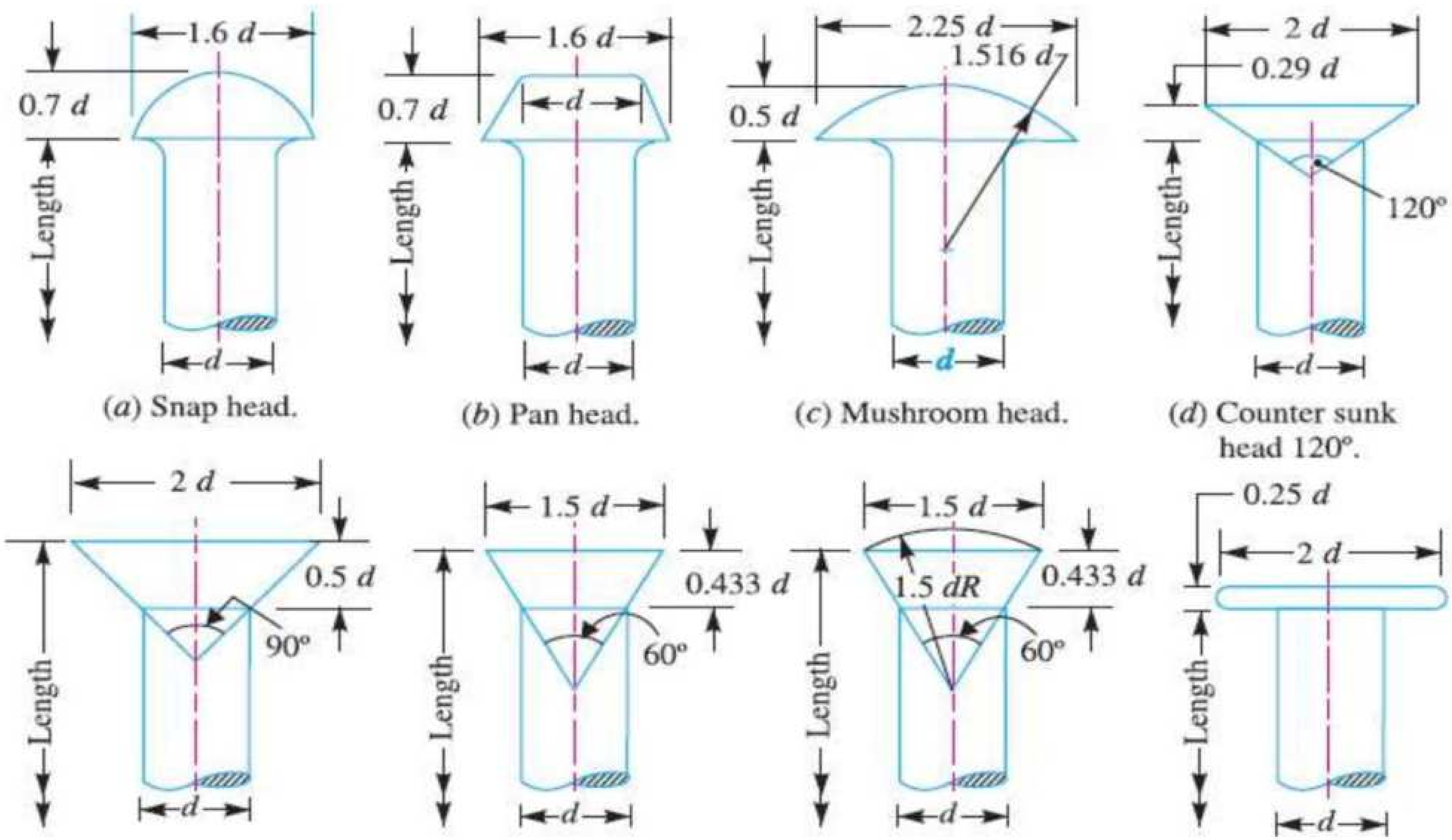
(b) Final position.

METHODS OF RIVETING

Types of Rivet Heads

According to Indian standard specifications, the rivet heads are classified into the following three types :

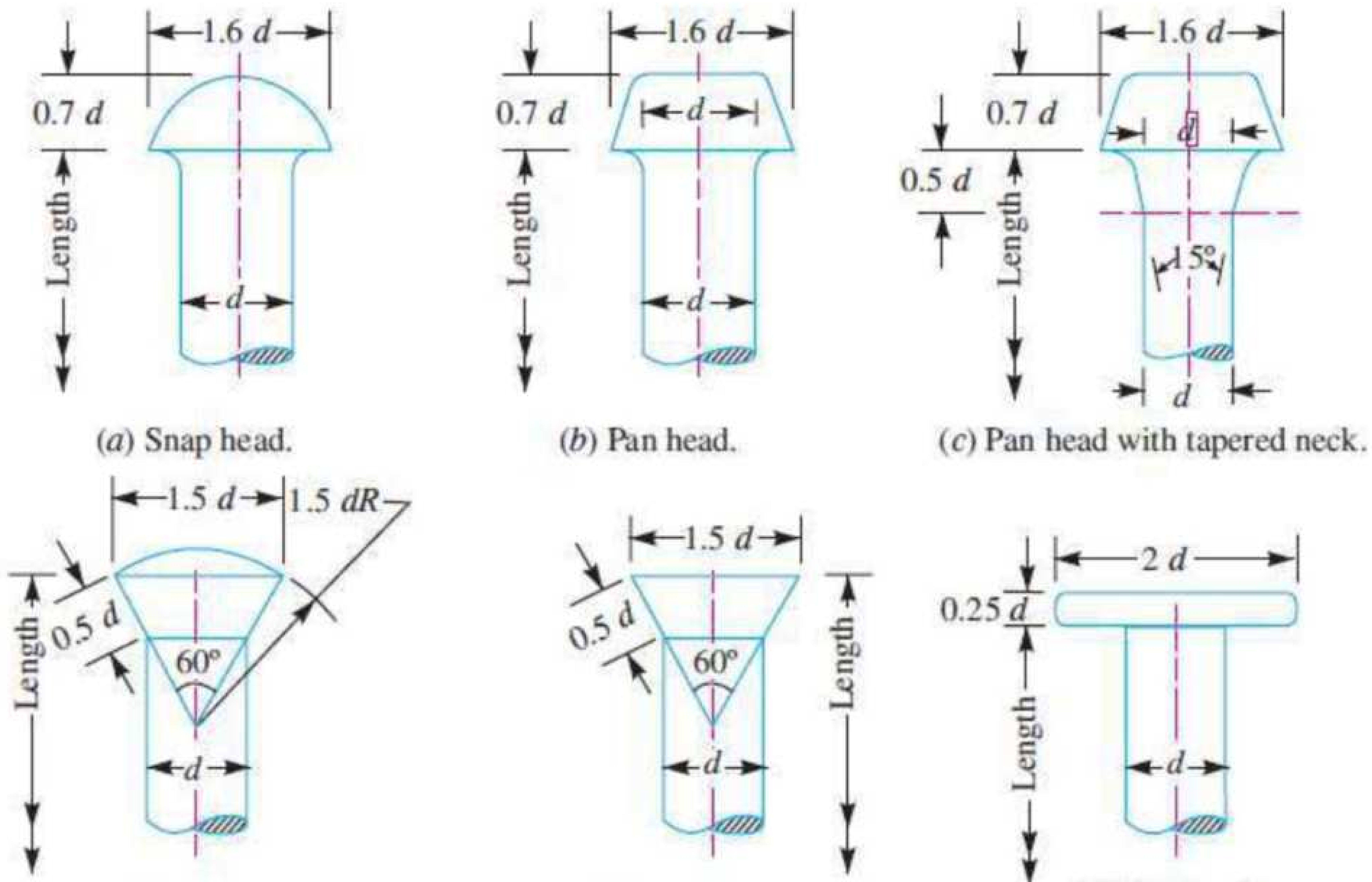
1. Rivet heads for general purposes (below 12 mm diameter) as shown in Fig., according to IS : 2155 – 1982 (Reaffirmed 1996).



(a) Snap head. (b) Pan head. (c) Mushroom head. (d) Counter sunk head 120°.

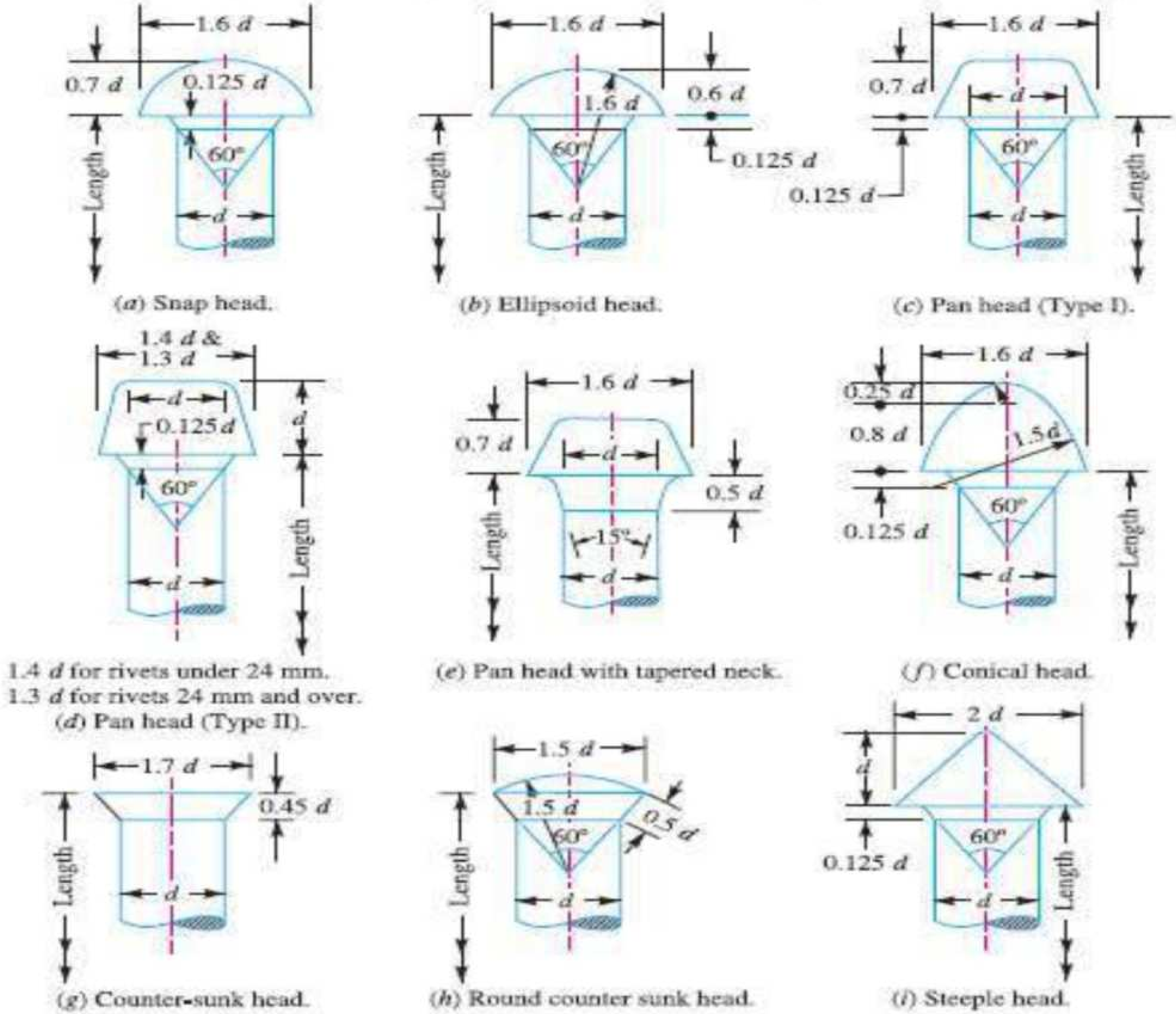
(e) Flat counter sunk head 90°. (f) Flat counter sunk head 60°. (g) Round counter sunk head 60°. (h) Flat head.

2. Rivet heads for general purposes (From 12 mm to 48 mm diameter) as shown in Fig., according to IS : 1929 – 1982 (Reaffirmed 1996).



(a) Snap head. (b) Pan head. (c) Pan head with tapered neck.
(d) Round counter sunk head 60°. (e) Flat counter sunk head 60°. (f) Flat head.

3. Rivet heads for boiler work (from 12 mm to 48 mm diameter, as shown in Fig. 9.5, according to IS : 1928 – 1961 (Reaffirmed 1996).



Important Terms Used in Riveted Joints

The following terms in connection with the riveted joints are important from the subject point of view :

1. **Pitch.** *It is the distance from the centre of one rivet to the centre of the next rivet measured* parallel to the seam as shown in Fig. It is usually denoted by ***p.***

2. **Back pitch.** *It is the perpendicular distance between the centre lines of the successive rows* as shown in Fig. It is usually denoted by ***pb.***

3. **Diagonal pitch.** *It is the distance between the centres of the rivets in adjacent rows of zig-zag* riveted joint as shown in Fig. It is usually denoted by ***pd.***

4. **Margin or marginal pitch.** *It is the distance between the centre of rivet hole to the nearest* edge of the plate as shown in Fig. It is usually denoted by ***m.***

Types of Riveted Joints

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

1. **Lap joint, and**
2. **Butt joint.**

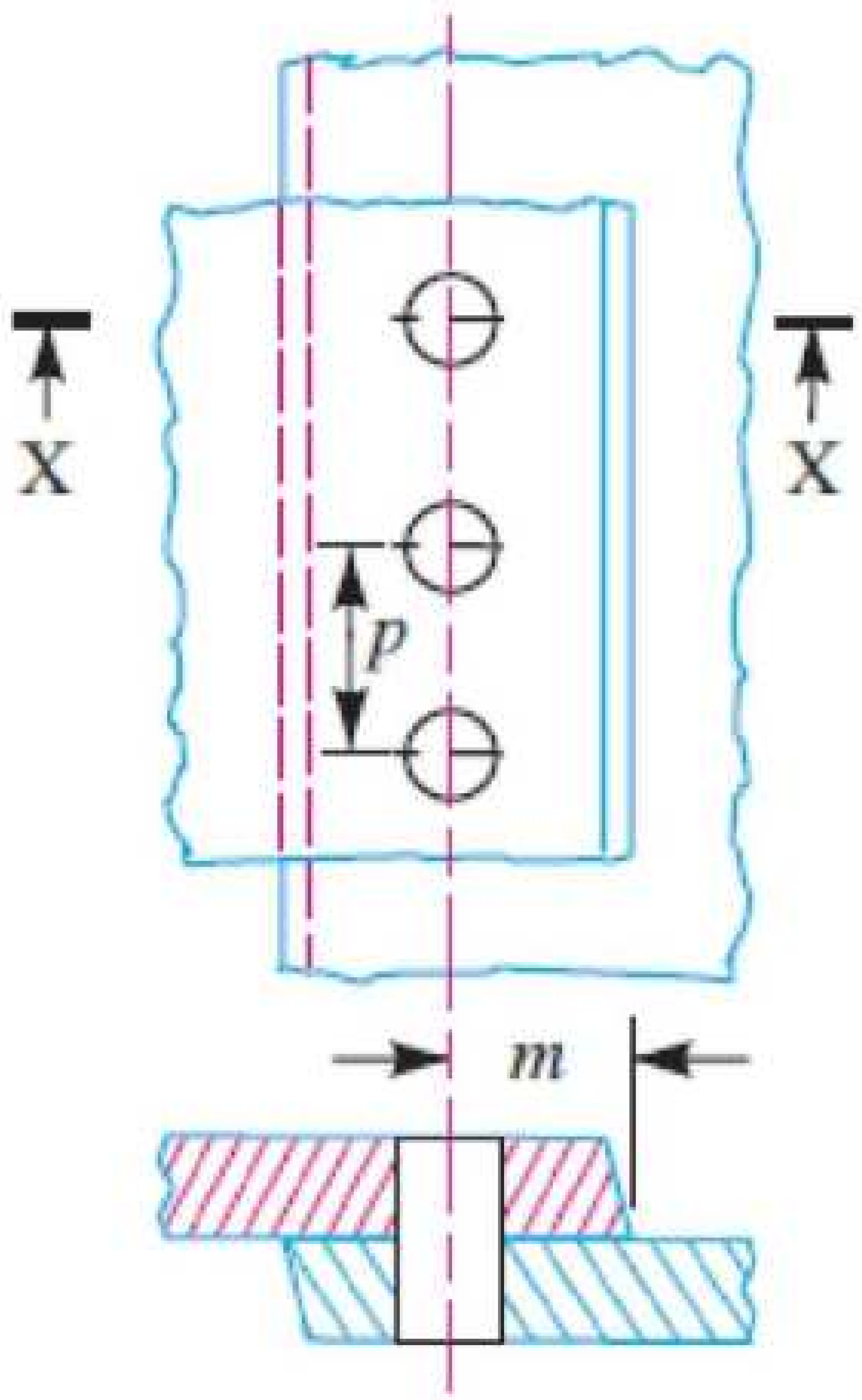
Lap Joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

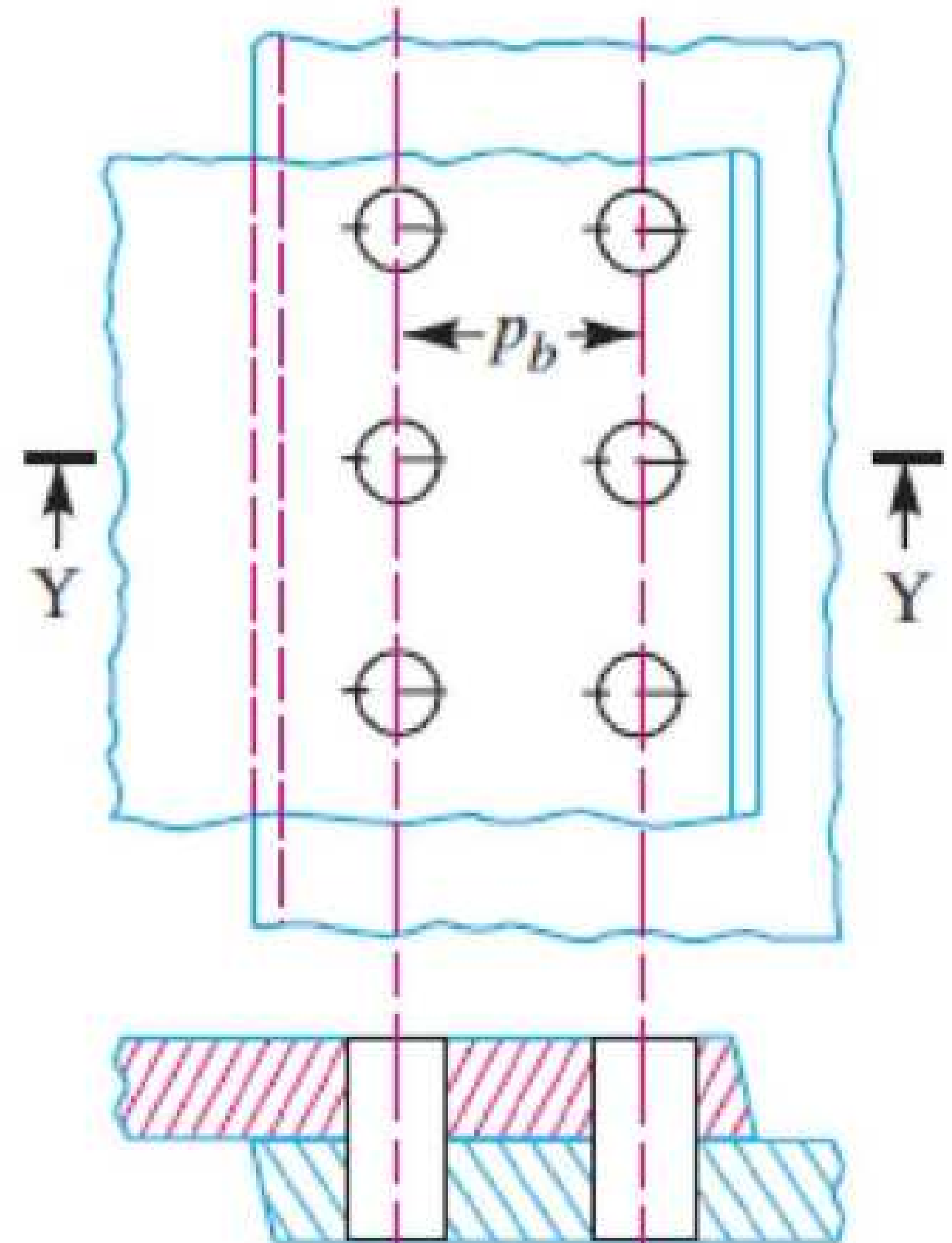
Butt Joint

A butt joint is that in which the main plates are kept in alignment butting (*i.e. touching) each other and a cover plate* (*i.e. strap) is placed either on one side or on both sides of the* main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types :

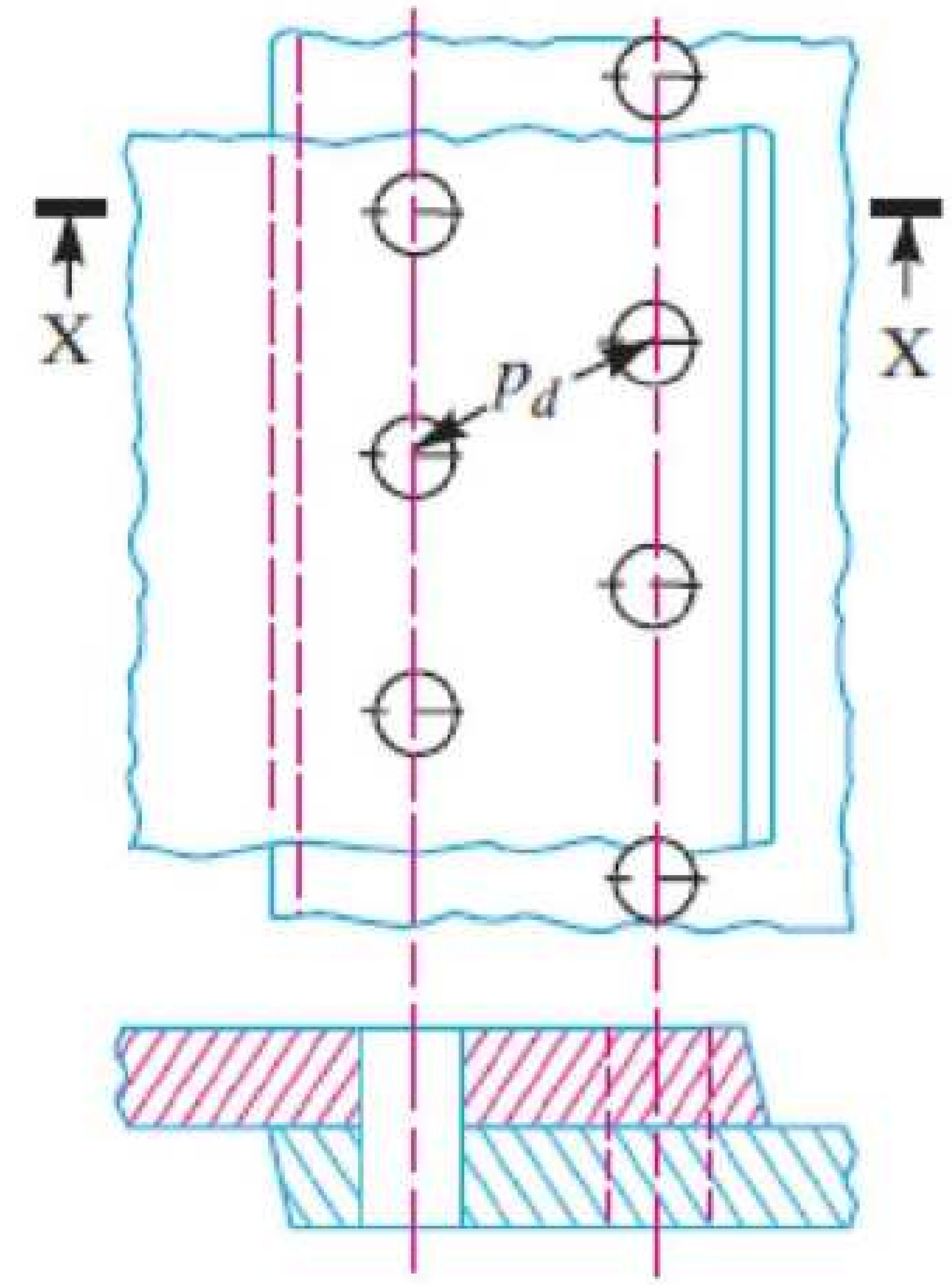
1. Single strap butt joint, and 2. Double strap butt joint.



(a) Single riveted lap joint.

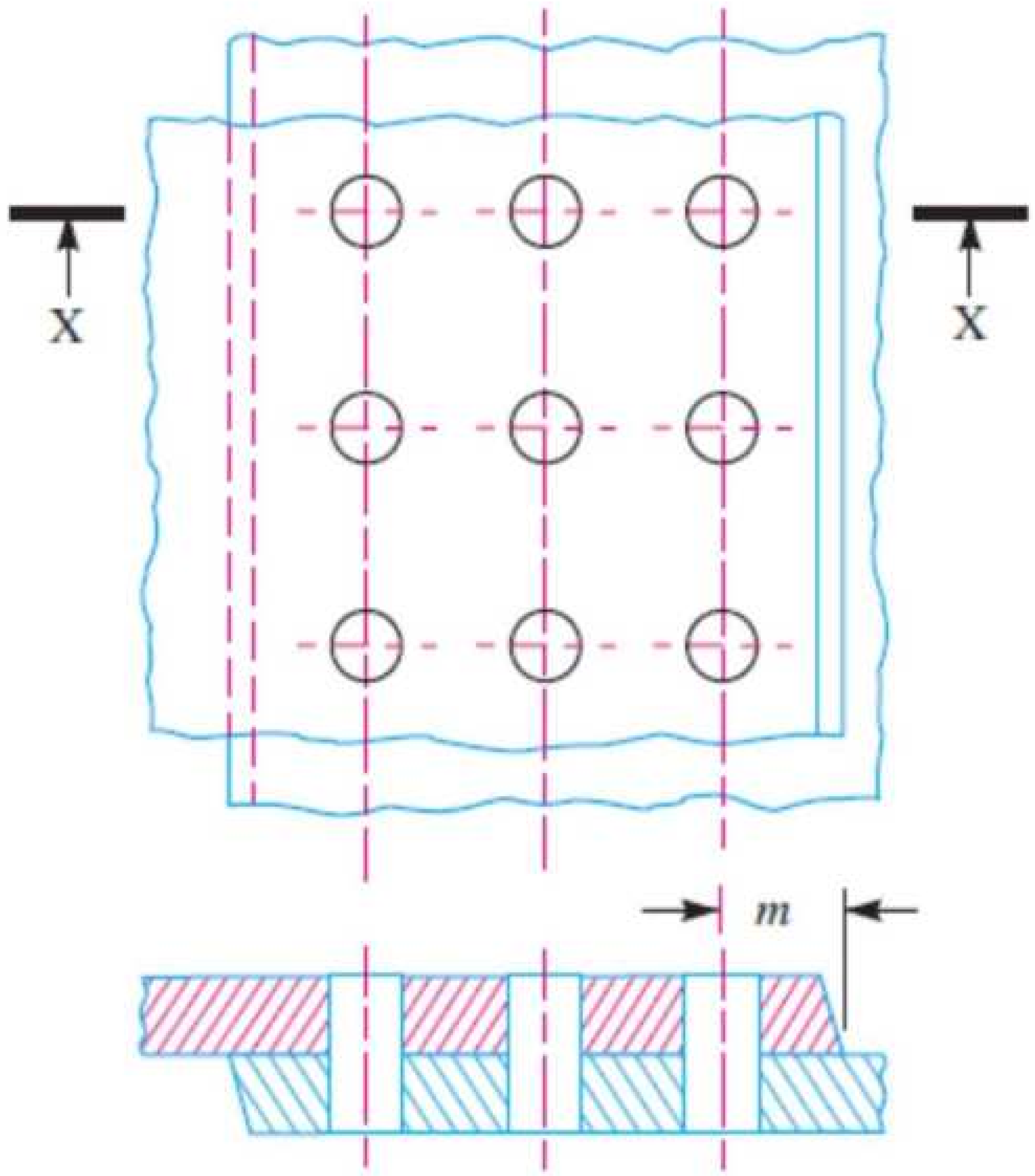


(b) Double riveted lap joint (Chain riveting).

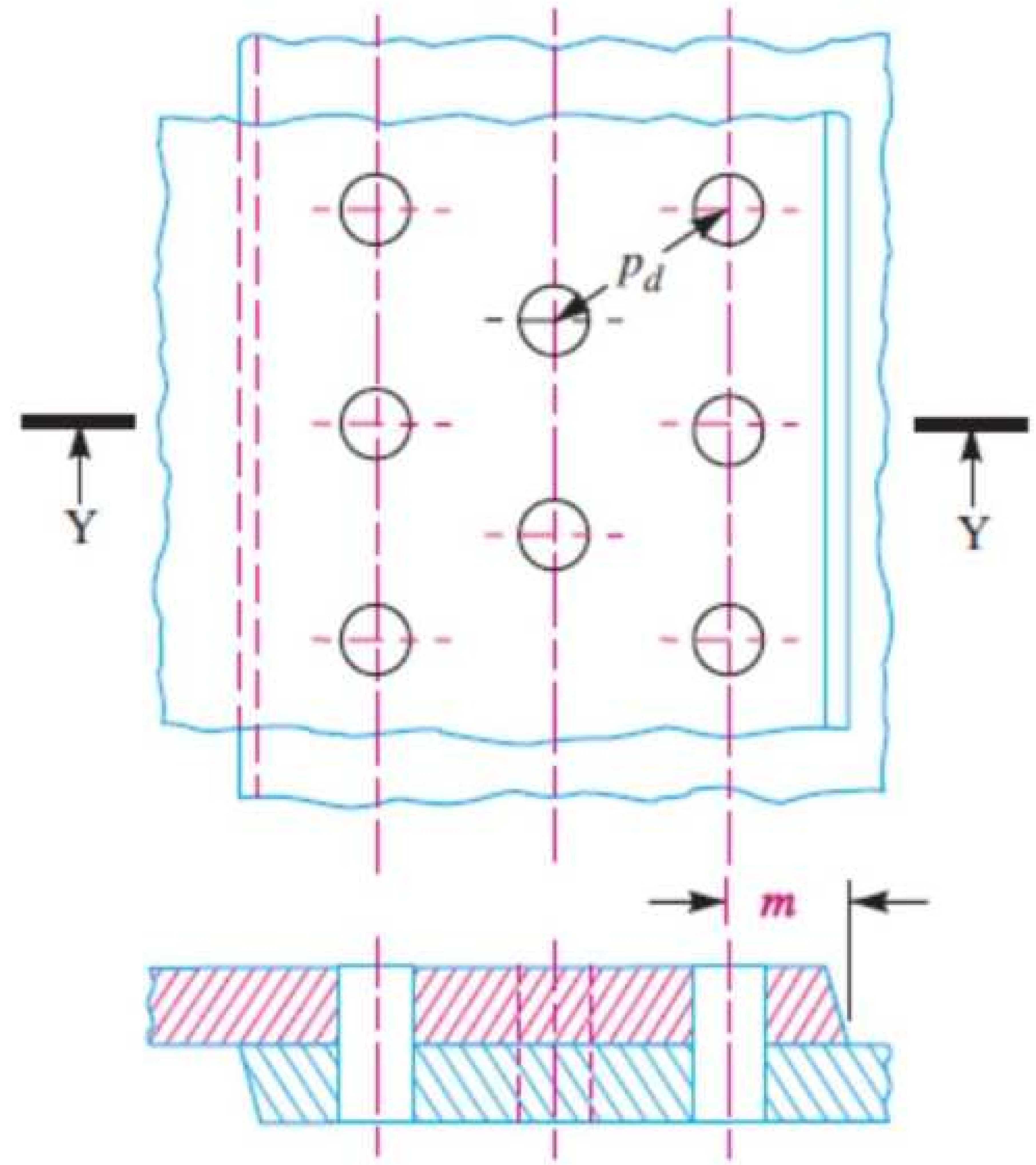


(c) Double riveted lap joint (Zig-zag riveting).

LAP JOINT



(a) Chain riveting.



(b) Zig-zag riveting.

TRIPLE RIVETED LAP JOINT

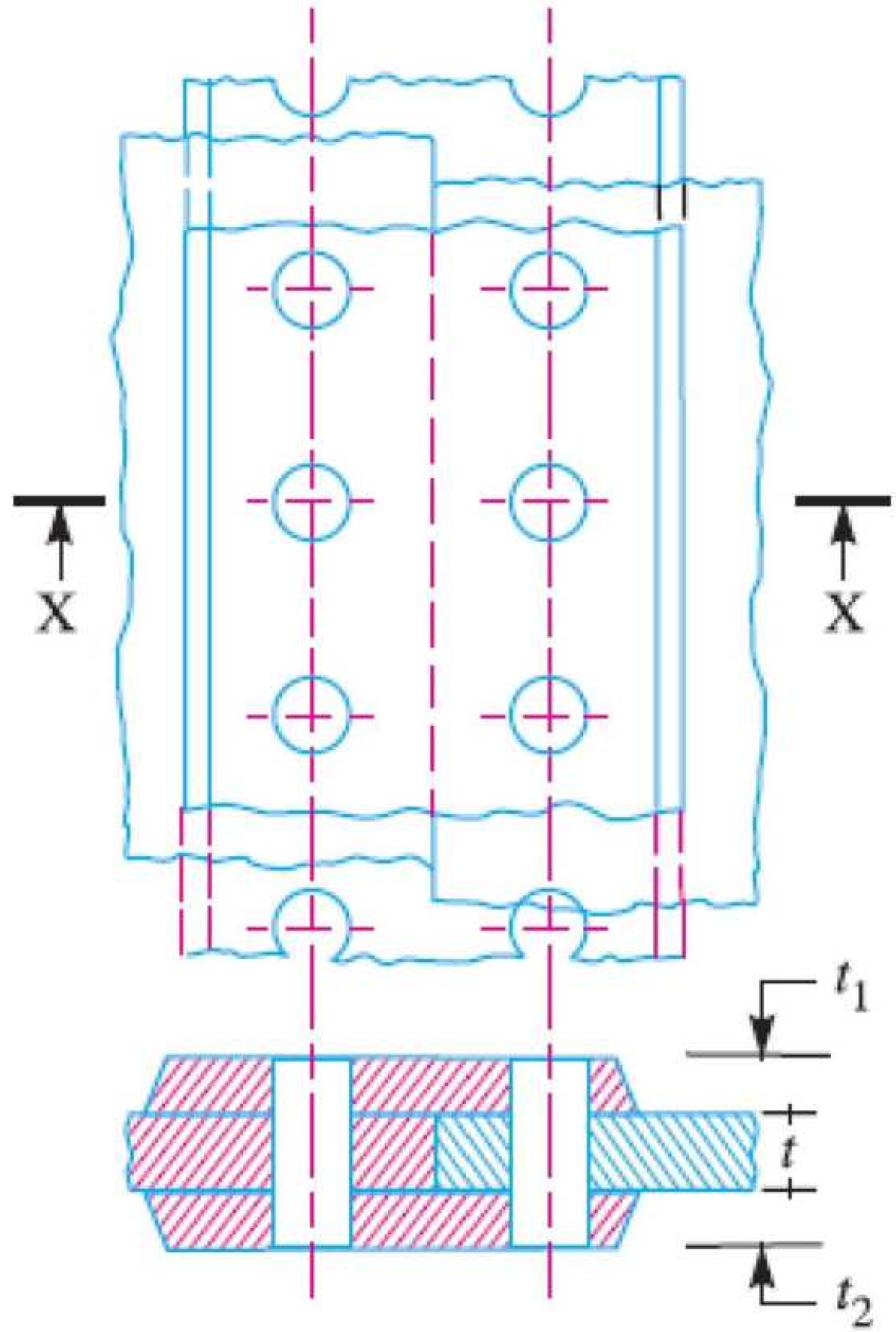
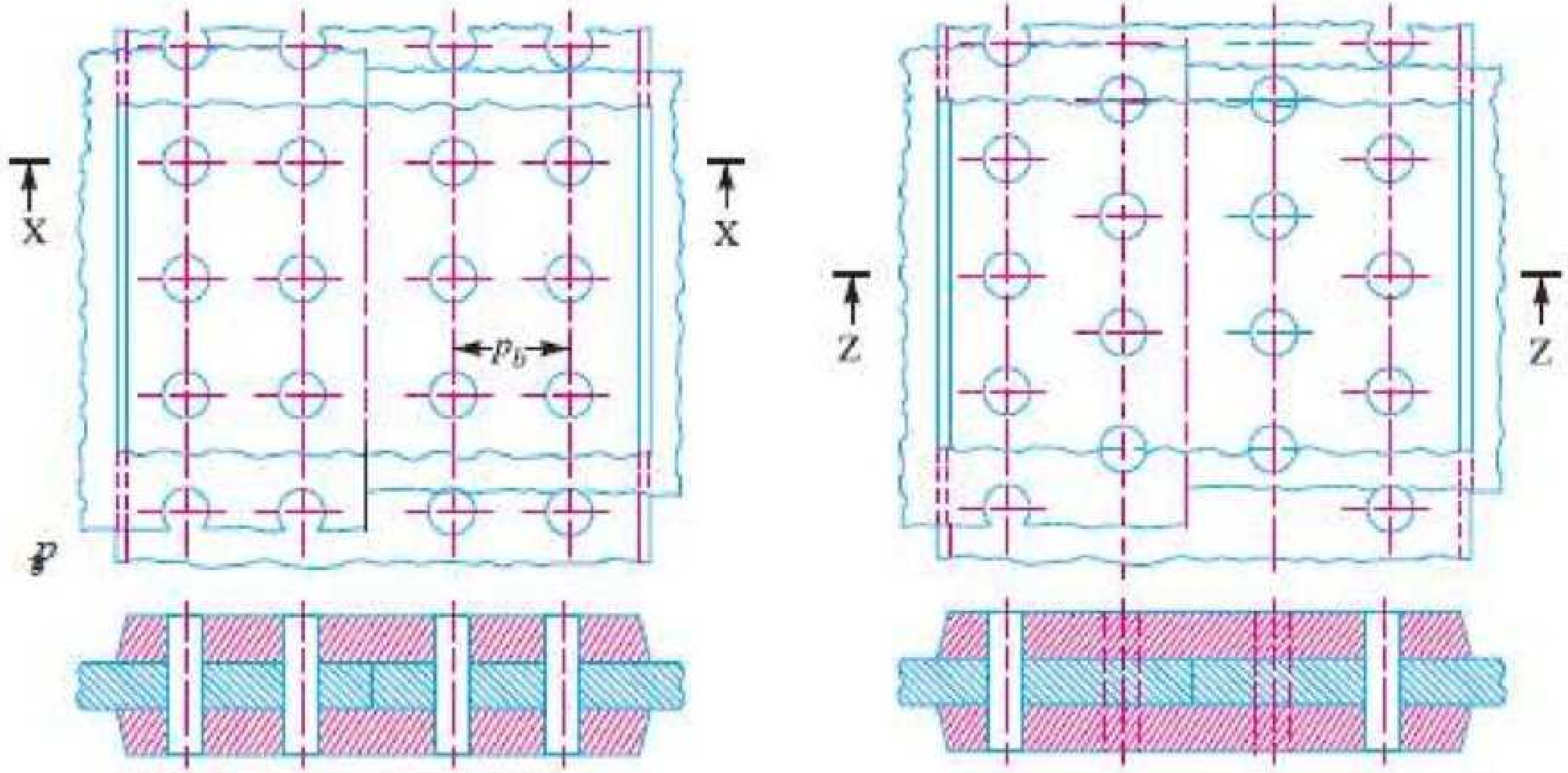


Fig. 9.8. Single riveted double strap butt joint.



(*a*) Chain riveting. (*b*) Zig-zag riveting.

Fig. 9.9. Double riveted double strap (equal) butt joints.

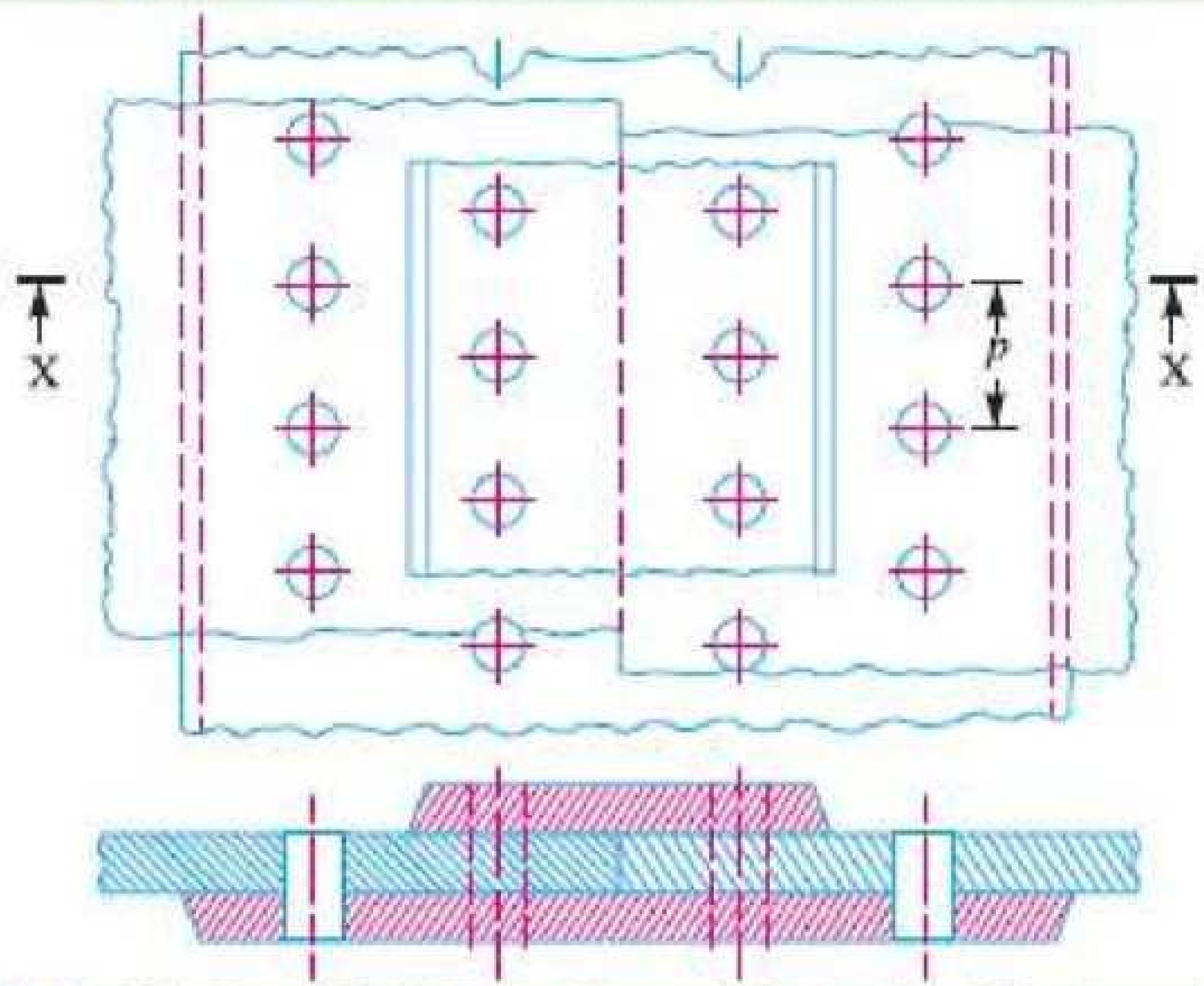


Fig. 9.10. Double riveted double strap (unequal) butt joint with zig-zag riveting.

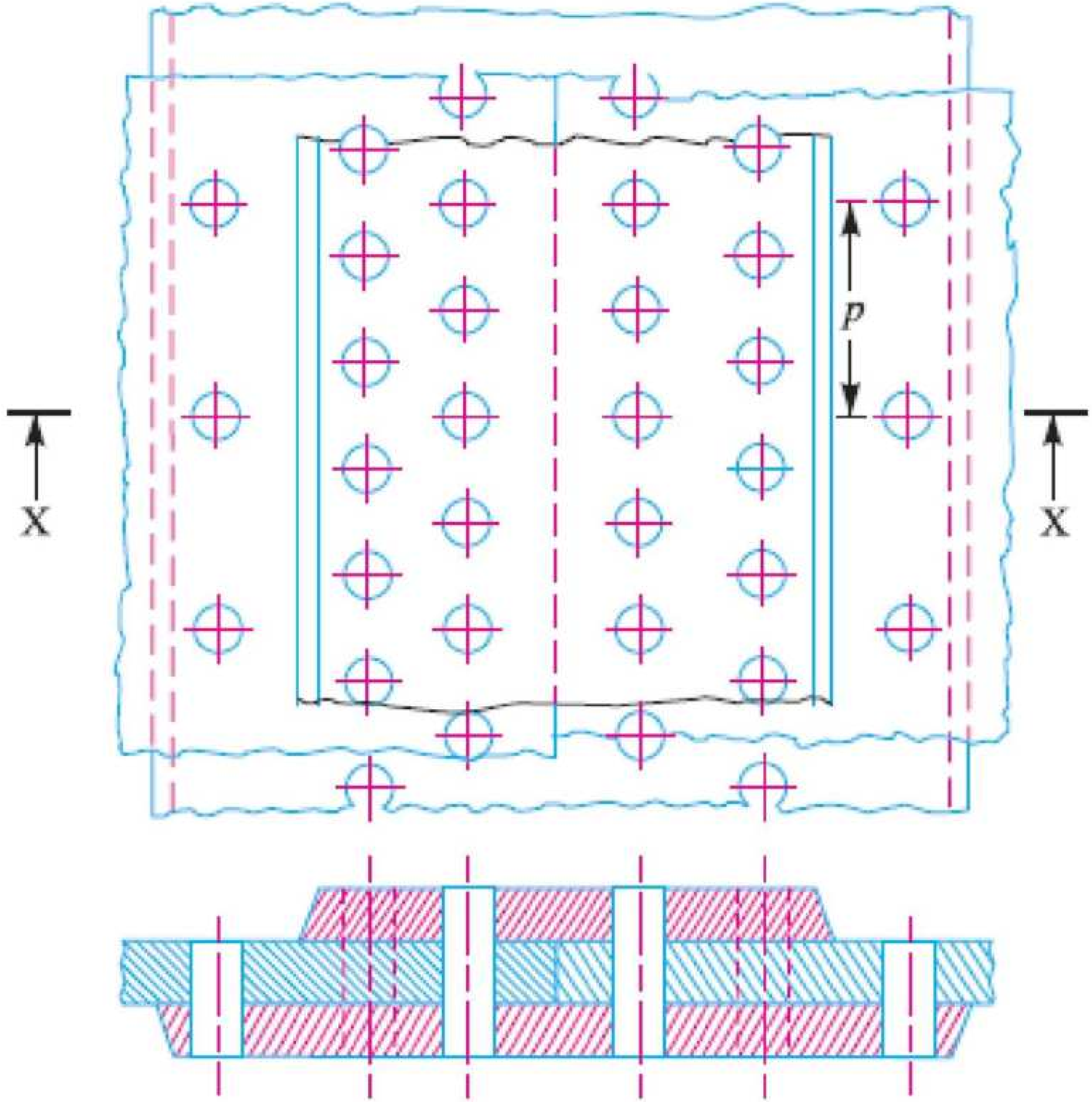


Fig. 9.11. Triple riveted double strap (unequal) butt joint.

Failures of a Riveted Joint

1. ***Tearing of the plate at an edge.*** *A joint may fail due to tearing of the plate at an edge as* shown in Fig. 9.13. This can be avoided by keeping the margin, $m = 1.5d$, *where d is the diameter of* the rivet hole.

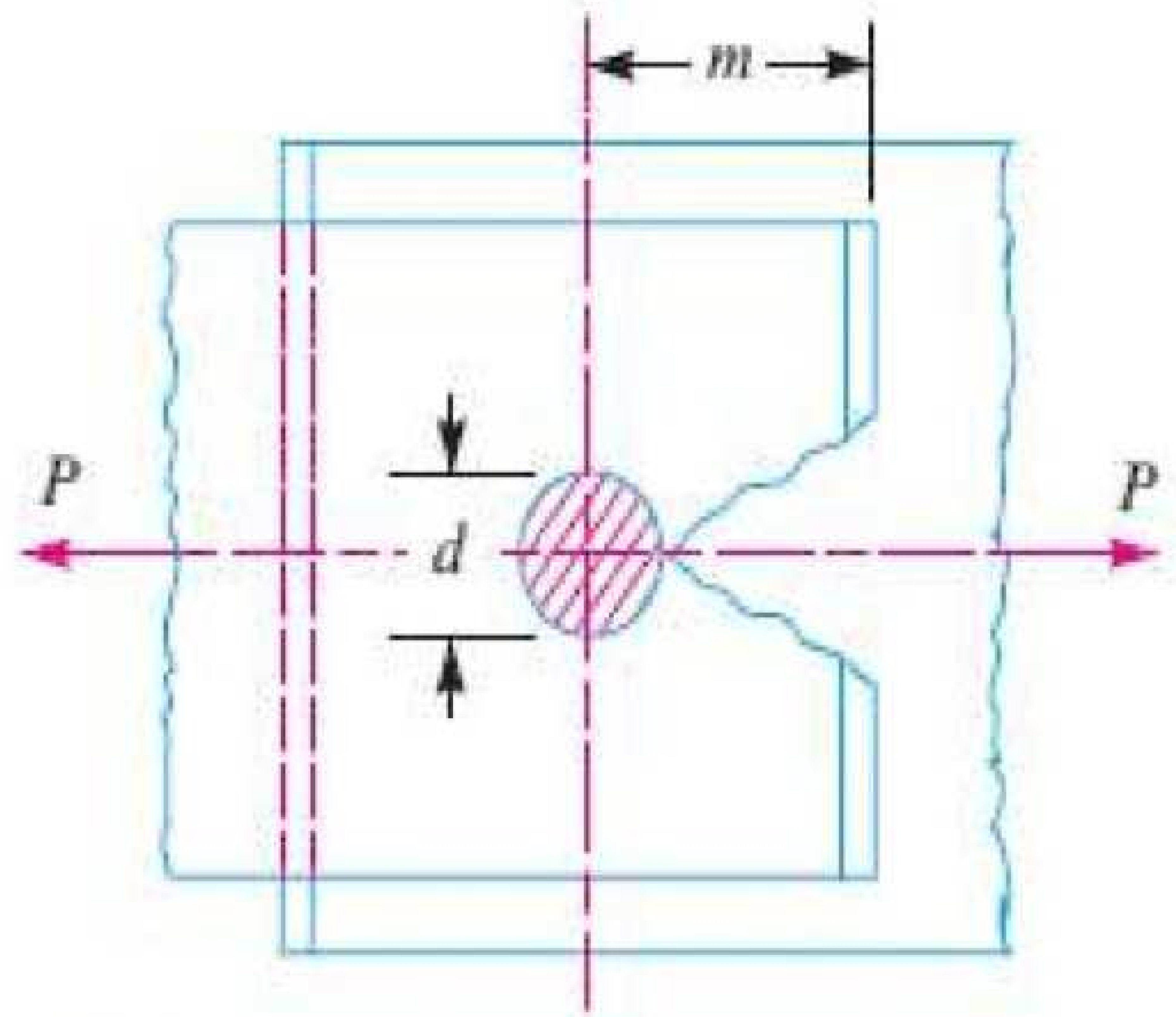


Fig. 9.13. Tearing of the plate at an edge.

2. *Tearing of the plate across a row of rivets. Due to the tensile stresses in the main plates, the* main plate or cover plates may tear off across a row of rivets as shown in Fig. 9.14. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

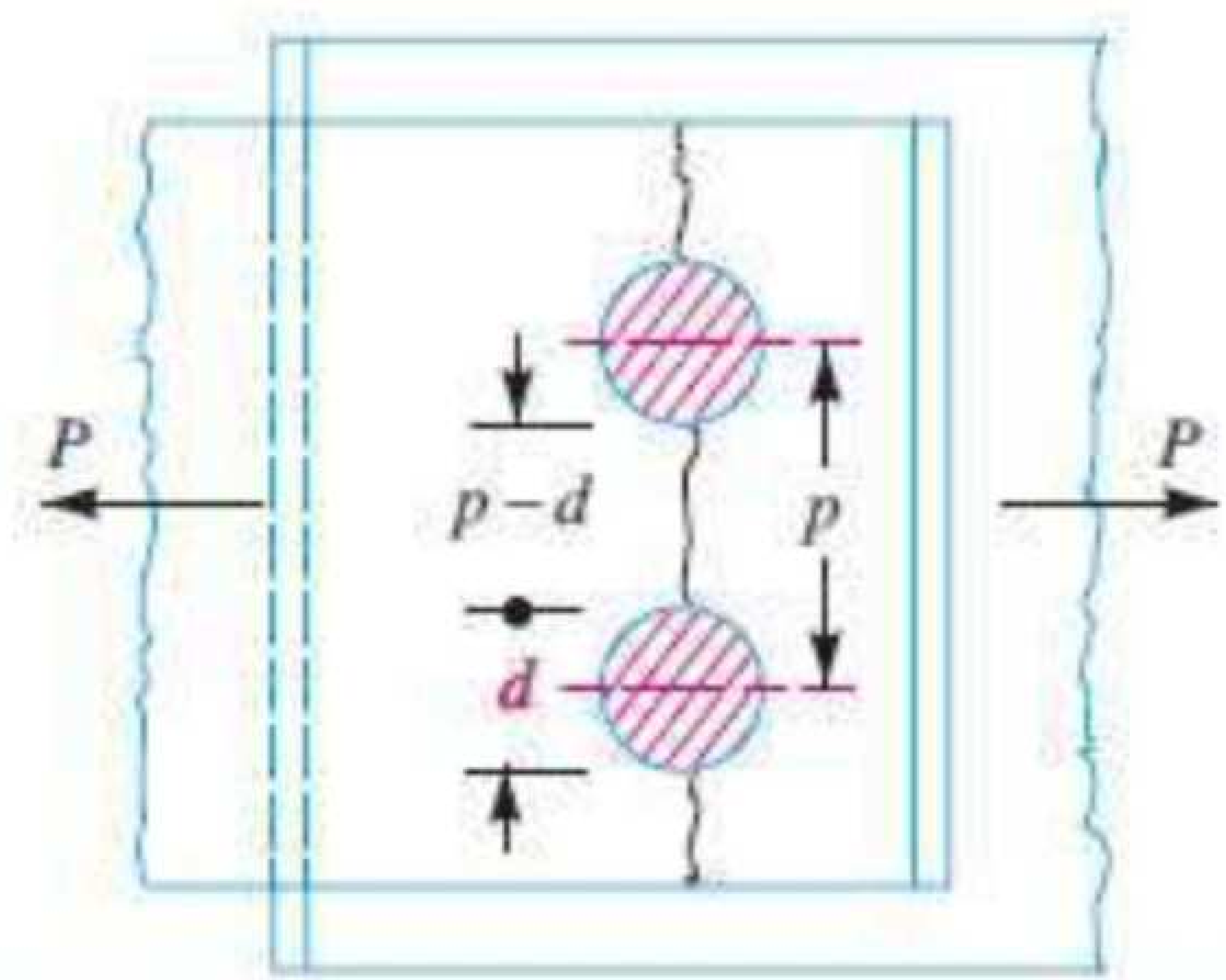


Fig. 9.14. Tearing of the plate across the rows of rivets.

The resistance offered by the plate against tearing is known as ***tearing resistance*** *or* ***tearing strength*** *or tearing value of the plate.*

Let p = *Pitch of the rivets,*

d = *Diameter of the rivet hole,*

t = *Thickness of the plate, and*

σt = *Permissible tensile stress for the plate material.*

We know that tearing area per pitch length,

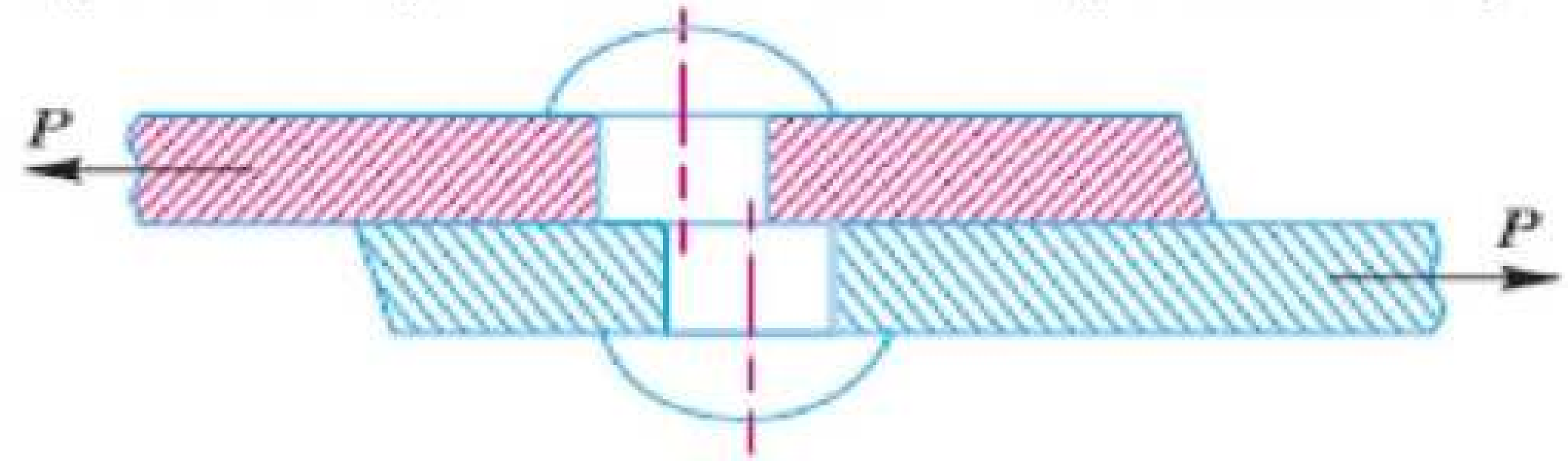
$$A_t = (p - d)\,t$$

∴ Tearing resistance or pull required to tear off the plate per pitch length,

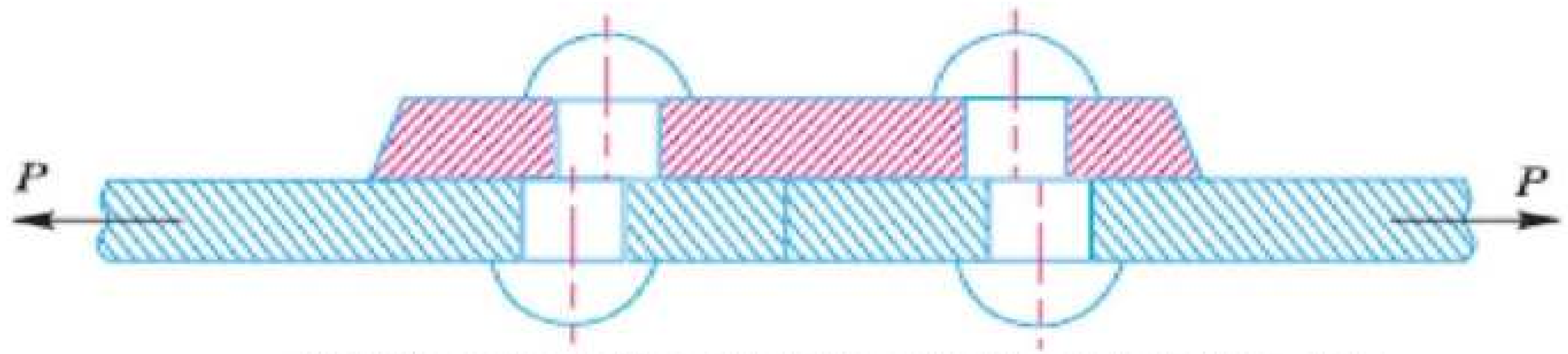
$$P_t = A_t.\sigma_t = (p - d)t.\sigma_t$$

When the tearing resistance (*Pt) is greater than the applied load (P) per pitch length, then this* type of failure will not occur.

3. *Shearing of the rivets.* *The plates which are connected by the rivets exert tensile stress on* the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig. 9.15. It may be noted that the rivets are in *single shear in a lap joint and in a single cover butt joint, as shown in Fig. 9.15. But the rivets are in double shear in a double cover butt joint as shown in Fig. 9.16. The resistance offered by a rivet to be sheared off is known as *shearing resistance or shearing strength or shearing value of the rivet.*



(*a*) Shearing off a rivet in a lap joint.



(*b*) Shearing off a rivet in a single cover butt joint.

Fig. 9.15. Shearing of rivets.

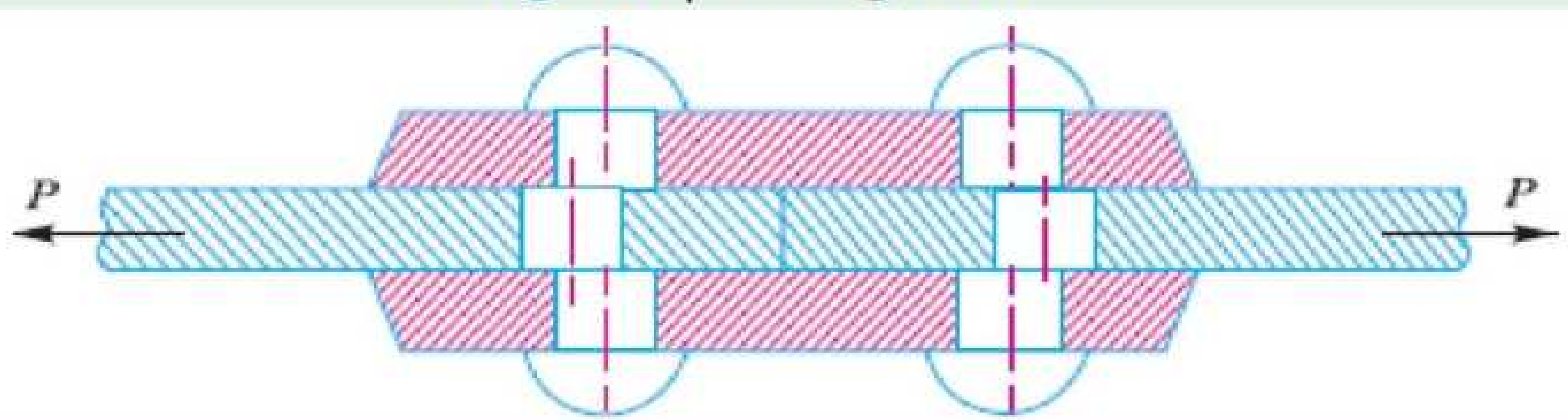


Fig. 9.16. Shearing off a rivet in double cover butt joint.

Let

d = *Diameter of the rivet hole,*

τ = Safe permissible shear stress for the rivet material, and

n = *Number of rivets per pitch length.*

We know that shearing area,

$A_s = (\pi/4) \times d^2$ ...*(In single shear)*

$= 2 \times (\pi/4) \times d^2$ ...*(Theoretically, in double shear)*

$= 1.875 \times (\pi/4) \times d^2$ ...*(In double shear, according to Indian Boiler Regulations)*

∴ Shearing resistance or pull required to shear off the rivet per pitch length,

$P_s = n \times (\pi/4) \times d^2 \times \tau$ ...*(In single shear)*

$= n \times 2 \times (\pi/4) \times d^2 \times \tau$ ...*(Theoretically, in double shear)*

$= n \times 1.875 \times (\pi/4) \times d^2 \times \tau$ ...*(In double shear, according to Indian Boiler Regulations)*

4. *Crushing of the plate or rivets.* *Sometimes, the rivets do not actually shear off under the* tensile stress, but are crushed as shown in Fig. 9.17. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as *bearing failure. The area which resists this action is the projected area of the hole or rivet on* diametral plane.

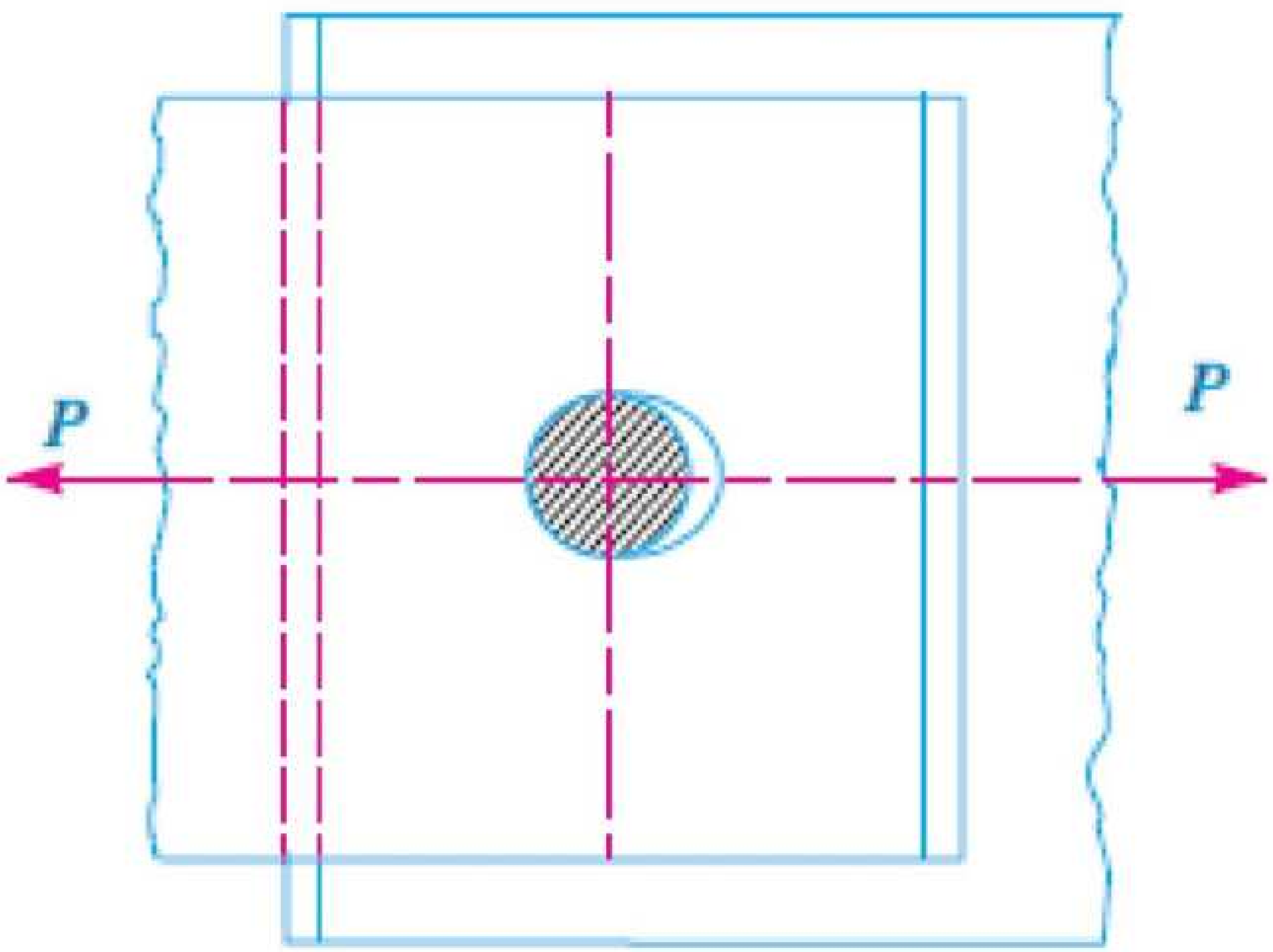


Fig. 9.17. Crushing of a rivet.

The resistance offered by a rivet to be crushed is known as *crushing resistance or crushing strength or bearing value of the rivet.*

Let d = *Diameter of the rivet hole,*
t = *Thickness of the plate,*
σ_c = *Safe permissible crushing stress for the rivet or* plate material, and
n = *Number of rivets per pitch length under crushing.*

We know that crushing area per rivet (*i.e. projected area per rivet),*

$$A_c = d.t$$

$$\therefore \text{Total crushing area} = n.d.t$$

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.d.t.\sigma_c$$

When the crushing resistance (*P_c) is greater than* the applied load (*P) per pitch length, then this type of* failure will occur.

Note : The number of rivets under shear shall be equal to the number of rivets under crushing.

Strength of a Riveted Joint

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail. We have seen that *Pt, Ps and Pc are the pulls required to tear off the plate*, shearing off the rivet and crushing off the rivet. A little consideration will show that if we go on increasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

If the joint is *continuous as in case of boilers, the strength is calculated per pitch length. But if* the joint is *small, the strength is calculated for the whole length of the plate.*

Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint

= Least of P_t, P_s and P_c

Strength of the un-riveted or solid plate per pitch length,

$$P = p \times t \times \sigma_t$$

∴ Efficiency of the riveted joint,

$$\eta = (\text{Least of } P_t, P_s \text{ and } P_c) / (p \times t \times \sigma_t)$$

where p = *Pitch of the rivets,*

t = *Thickness of the plate, and*

σ_t = *Permissible tensile stress of the plate material.*

Example 1. *A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.*

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution.

Given : $t = 15$ mm ; $d = 25$ mm ; $p = 75$ mm ; $\sigma_{tu} = 400$ MPa = 400 N/mm2 ; $\tau_u = 320$ MPa = 320 N/mm2 ; $\sigma_{cu} = 640$ MPa = 640 N/mm2

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma tu = (75 - 25)15 \times 400 = 300\ 000\ N$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times 4\pi \times d\ 2 \times \tau u = 2 \times 4\pi(25)2\ 320 = 314\ 200\ \text{N} \ ...(\text{Q}\ n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma cu = 2 \times 25 \times 15 \times 640 = 480\ 000\ N$$

From above we see that the minimum force per pitch which will rupture the joint is 300000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint = 300000/4 = 75000 N

Let σ_{ta}, τ_a *and* σ_{ca} *be the actual tearing, shearing and crushing stresses produced with a safe* load of 75000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (*Pta)*,

$$75\,000 = (p - d)\,t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\,\sigma_{ta}$$

$$\therefore \sigma_{ta} = 75\,000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa } \textbf{Ans.}$$

Actual shearing resistance of the rivets (*Psa)*,

$$75\,000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4}(25)^2\,\tau_a = 982\,\tau_a$$

$$\therefore \tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa } \textbf{Ans.}$$

and actual crushing resistance of the rivets (*Pca)*,

$$75\,000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\,\sigma_{ca}$$

$$\therefore \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa } \textbf{Ans.}$$

Assumptions in Designing Boiler Joints

The following assumptions are made while designing a joint for boilers :

1. The load on the joint is equally shared by all the rivets. The assumption implies that the shell and plate are rigid and that all the deformation of the joint takes place in the rivets themselves.
2. The tensile stress is equally distributed over the section of metal between the rivets.
3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Boiler

1. **Thickness of boiler shell.** ***First of all, the thickness of the boiler shell is determined by using*** the thin cylindrical formula,

i.e. $t = \left(\frac{P \cdot D}{2\,\sigma_t \times \eta_l}\right) + 1$ mm as corrosion allowance

where t = *Thickness of the boiler shell,*

P = *Steam pressure in boiler,*

D = *Internal diameter of boiler shell,*

σ_t = *Permissible tensile stress, and*

η_l = *Efficiency of the longitudinal joint.*

The following points may be noted :

(a) The thickness of the boiler shell should not be less than 7 mm.

(b) The efficiency of the joint may be taken from the following table.

Table 9.1. Efficiencies of commercial boiler joints.

Lap joints	*Efficiency (%)*	**Maximum efficiency*	*Butt joints (Double strap)*	*Efficiency (%)*	**Maximum efficiency*
Single riveted	45 to 60	63.3	Single riveted	55 to 60	63.3
Double riveted	63 to 70	77.5	Double riveted	70 to 83	86.6
Triple riveted	72 to 80	86.6	Triple riveted (5 rivets per pitch with unequal width of straps)	80 to 90	95.0
			Quadruple riveted	85 to 94	98.1

* The maximum efficiencies are valid for ideal equistrength joints with tensile stress = 77 MPa, shear stress = 62 MPa and crushing stress = 133 MPa.

Indian Boiler Regulations (I.B.R.) allow a maximum efficiency of 85% for the best joint.

(*c*) According to I.B.R., the factor of safety should not be less than 4. The following table shows the values of factor of safety for various kind of joints in boilers.

Table 9.2. Factor of safety for boiler joints.

Type of joint	*Factor of safety*	
	Hand riveting	*Machine riveting*
Lap joint	4.75	4.5
Single strap butt joint	4.75	4.5
Single riveted butt joint with two equal cover straps	4.75	4.5
Double riveted butt joint with two equal cover straps	4.25	4.0

2. Diameter of rivets. *After finding out the thickness of the boiler shell (t), the diameter of the* rivet hole *(d) may be determined by using Unwin's empirical formula, i.e.*

$$d = 6\sqrt{t} \text{ (when } t \text{ is greater than 8 mm)}$$

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing. The following table gives the rivet diameter corresponding to the diameter of rivet hole as per IS : 1928 – 1961 (Reaffirmed 1996).

Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).

Basic size of rivet mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole diameter (min) mm	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

According to IS : 1928 – 1961 (Reaffirmed 1996), the table on the next page (Table 9.4) gives the preferred length and diameter combination for rivets.

3. *Pitch of rivets.* *The pitch of the rivets is obtained by equating the tearing resistance of the* plate to the shearing resistance of the rivets. It may noted that

(a) *The pitch of the rivets should not be less than 2d, which is necessary for the formation* of head.

(b) *The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per* I.B.R. is

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where

t = *Thickness of the shell plate in mm, and*

C = *Constant.*

The value of the constant *C is given in Table 9.5.*

Table 9.4. Preferred length and diameter combinations for rivets used in boilers as per IS : 1928–1961 (Reaffirmed 1996).

(All dimensions in mm)

Length	*Diameter*													
	12	14	16	18	20	22	24	27	30	33	36	39	42	48
28	×	–	–	–	–	–	–	–	–	–	–	–	–	–
31.5	×	×	–	–	–	–	–	–	–	–	–	–	–	–
35.5	×	×	×	–	–	–	–	–	–	–	–	–	–	–
40	×	×	×	×	–	–	–	–	–	–	–	–	–	–
45	×	×	×	×	×	–	–	–	–	–	–	–	–	–
50	×	×	×	×	×	×	–	–	–	–	–	–	–	–
56	×	×	×	×	×	×	×	–	–	–	–	–	–	–
63	×	×	×	×	×	×	×	×	–	–	–	–	–	–
71	×	×	×	×	×	×	×	×	×	–	–	–	–	–
80	×	×	×	×	×	×	×	×	×	–	–	–	–	–
85	–	×	×	×	×	×	×	×	×	×	–	–	–	–
90	–	×	×	×	×	×	×	×	×	×	–	–	–	–
95	–	×	×	×	×	×	×	×	×	×	×	–	–	–
100	–	–	×	×	×	×	×	×	×	×	×	–	–	–
106	–	–	×	×	×	×	×	×	×	×	×	×	–	–
112	–	–	×	×	×	×	×	×	×	×	×	×	–	–
118	–	–	–	×	×	×	×	×	×	×	×	×	×	–
125	–	–	–	–	×	×	×	×	×	×	×	×	×	×
132	–	–	–	–	–	×	×	×	×	×	×	×	×	×
140	–	–	–	–	–	×	×	×	×	×	×	×	×	×
150	–	–	–	–	–	–	×	×	×	×	×	×	×	×
160	–	–	–	–	–	–	×	×	×	×	×	×	×	×
180	–	–	–	–	–	–	–	×	×	×	×	×	×	×
200	–	–	–	–	–	–	–	–	×	×	×	×	×	×
224	–	–	–	–	–	–	–	–	–	×	×	×	×	×
250	–	–	–	–	–	–	–	–	–	–	–	–	×	×

Preferred numbers are indicated by ×.

Table 9.5. Values of constant C.

Number of rivets per pitch length	*Lap joint*	*Butt joint (single strap)*	*Butt joint (double strap)*
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Note : If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than p_{max}, *then the value of* p_{max} *is taken.*

4. *Distance between the rows of rivets.* *The distance between the rows of rivets as specified by* Indian Boiler Regulations is as follows :

(*a*) *For equal number of rivets in more than one row for lap joint or butt joint, the distance* between the rows of rivets (*pb*) *should not be less than*

$$0.33\,p + 0.67\,d, \text{ for zig-zig riveting, and}$$

$$2\,d, \text{ for chain riveting.}$$

(*b*) *For joints in which the number of rivets in outer rows is half the number of rivets in inner* rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than

$$0.33\,p + 0.67 \text{ or } 2\,d, \text{ whichever is greater.}$$

The distance between the rows in which there are full number of rivets shall not be less than 2*d*.

(*c*) *For joints in which the number of rivets in outer rows is half the number of rivets in inner* rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than $0.2\,p + 1.15\,d$. *The distance between the rows in which* there are full number of rivets (zig-zag) shall not be less than $0.165\,p + 0.67\,d$.

Note : In the above discussion, *p is the pitch of the rivets in the outer rows*.

5. *Thickness of butt strap.* According to I.B.R., the thicknesses for butt strap (t_1) are as given below :

(*a*) The thickness of butt strap, in no case, shall be less than 10 mm.

(*b*) $t_1 = 1.125\, t$, for ordinary (chain riveting) single butt strap.

$t_1 = 1.125\, t \left(\dfrac{p-d}{p-2d}\right)$, for single butt straps, every alternate rivet in outer rows being omitted.

$t_1 = 0.625\, t$, for double butt-straps of equal width having ordinary riveting (chain riveting).

$t_1 = 0.625\, t \left(\dfrac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(*c*) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75\, t$, for wide strap on the inside, and

$t_2 = 0.625\, t$, for narrow strap on the outside.

6. *Margin.* The margin (m) is taken as $1.5\, d$.

Note : The above procedure may also be applied to ordinary riveted joints.

DESIGN OF SHAFTS AND KEYS

INTRODUCTION

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

MATERIAL USED FOR SHAFTS

The material used for shafts should have the following properties :

1. **It should have high strength.**

2. **It should have good machinability.**

3. **It should have low notch sensitivity factor.**

4. **It should have good heat treatment properties.**

5. **It should have high wear resistant properties.**

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

DESIGN OF SHAFTS

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(*a*) *Shafts subjected to twisting moment or torque only,*

(*b*) *Shafts subjected to bending moment only,*

(*c*) *Shafts subjected to combined twisting and bending moments, and*

(*d*) *Shafts subjected to axial loads in addition to combined torsional and bending loads.*

SHAFTS SUBJECTED TO TWISTING MOMENT ONLY

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft

may be obtained by using the torsion equation. We know that

$$T/J = \tau/r \ldots(i)$$

where T = *Twisting moment (or torque) acting upon the shaft,*

J = *Polar moment of inertia of the shaft about the axis of rotation,*

τ = Torsional shear stress, and

r = *Distance from neutral axis to the outer most fibre*

= *d / 2; where d is the diameter of the shaft.*

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...(ii)}$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...(iii)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \text{...(iv)}$$

From the equations (*iii*) or (*iv*), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2 \pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2 \pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2) R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : $N = 200$ r.p.m. ; $P = 20 \text{ kW} = 20 \times 10^3 \text{ W}$; $\tau = 42 \text{ MPa} = 42 \text{ N/mm}^2$

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\,733 \text{ or } d = 48.7 \text{ say } 50 \text{ mm}$$ **Ans.**

Example 2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm²

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

∴ $$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or $$d = 159.4 \text{ say } 160 \text{ mm}$$ **Ans.**

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$M / I = \sigma b / y \qquad \ldots(i)$$

where M = *Bending moment,*

I = *Moment of inertia of cross-sectional area of the shaft about the* axis of rotation,

σb = *Bending stress, and*

y = *Distance from neutral axis to the outer-most fibre.*

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \qquad \text{...(where } k = d_i / d_o\text{)}$$

and $$y = d_o / 2$$

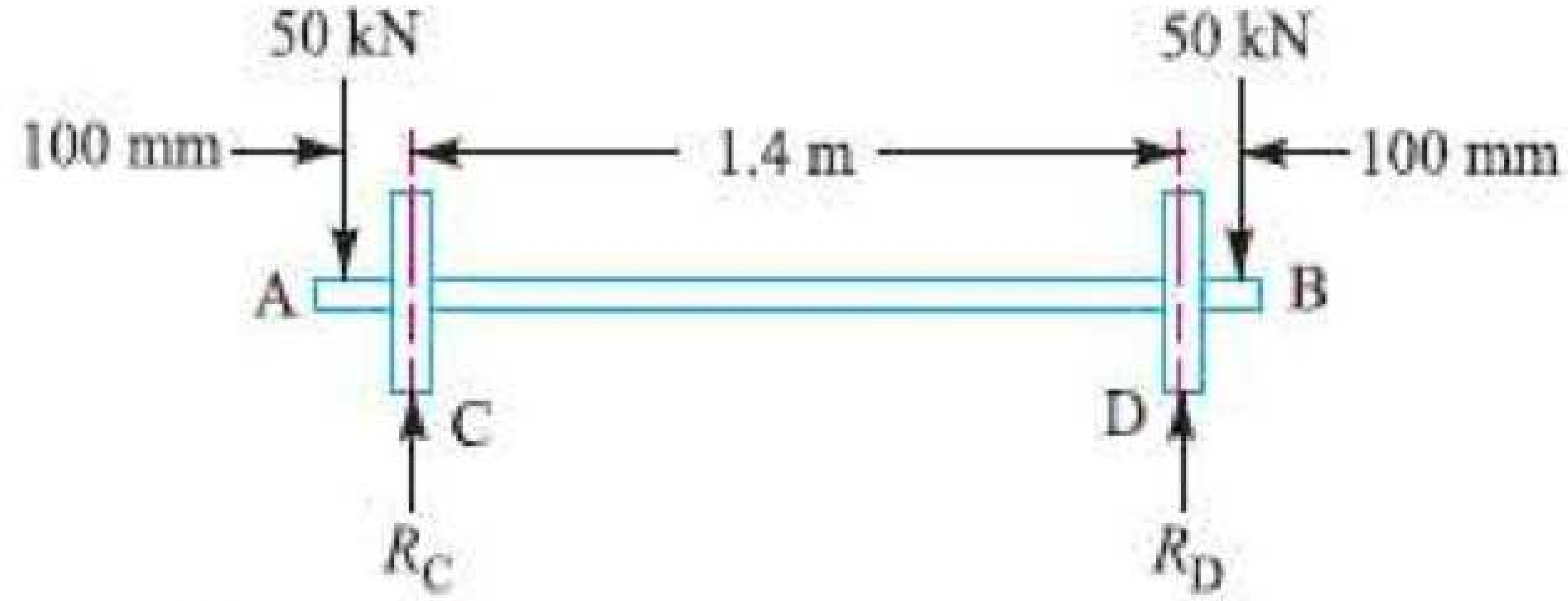
Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Example 1. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50 \text{ kN} = 50 \times 10^3 \text{ N}$; $L = 100 \text{ mm}$; $x = 1.4 \text{ m}$; $\sigma_b = 100 \text{ MPa} = 100 \text{ N/mm}^2$



The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = WL = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (*M*),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$\therefore$ $d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$ say 80 mm **Ans.**

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or
$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as *equivalent twisting moment* and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation *(i)* may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or
$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M+\sqrt{M^2+T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress $[\sigma_{b(max)}]$ equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M+\sqrt{M^2+T^2}\right] = \frac{\pi}{32}\times\sigma_b\times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: **1.** In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2+T^2} = \frac{\pi}{16}\times\tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \tfrac{1}{2}\left(M+\sqrt{M^2+T^2}\right) = \frac{\pi}{32}\times\sigma_b\,(d_o)^3\,(1-k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 1. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm² ; $\tau_u = 500$ MPa $= 500$ N/mm²

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$\therefore$ $d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

Taking the larger of the two values, we have

$d = 86$ say 90 mm Ans.

KEYS

Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys, 2. Saddle keys, 3. Tangent keys, 4. Round keys, and 5. Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. *Rectangular sunk key.* A rectangular sunk key is shown in Fig. The usual proportions of this key are :

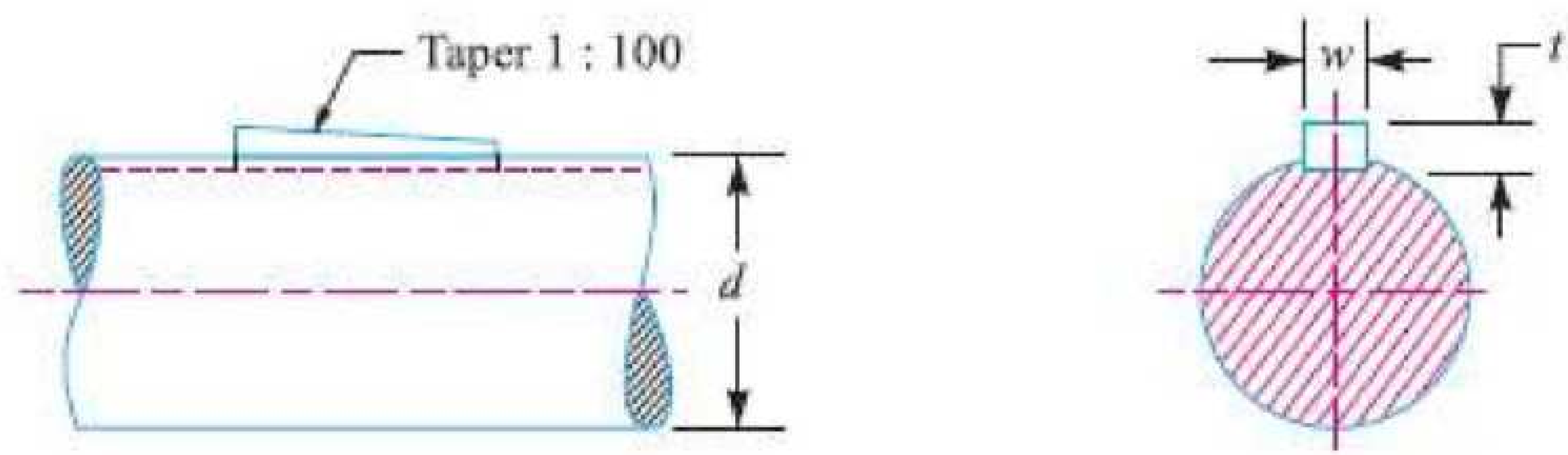
Width of key, *w = d / 4 ;*

and thickness of key, t = 2w / 3 = d / 6

where

d = Diameter of the shaft or diameter of the hole in the hub.

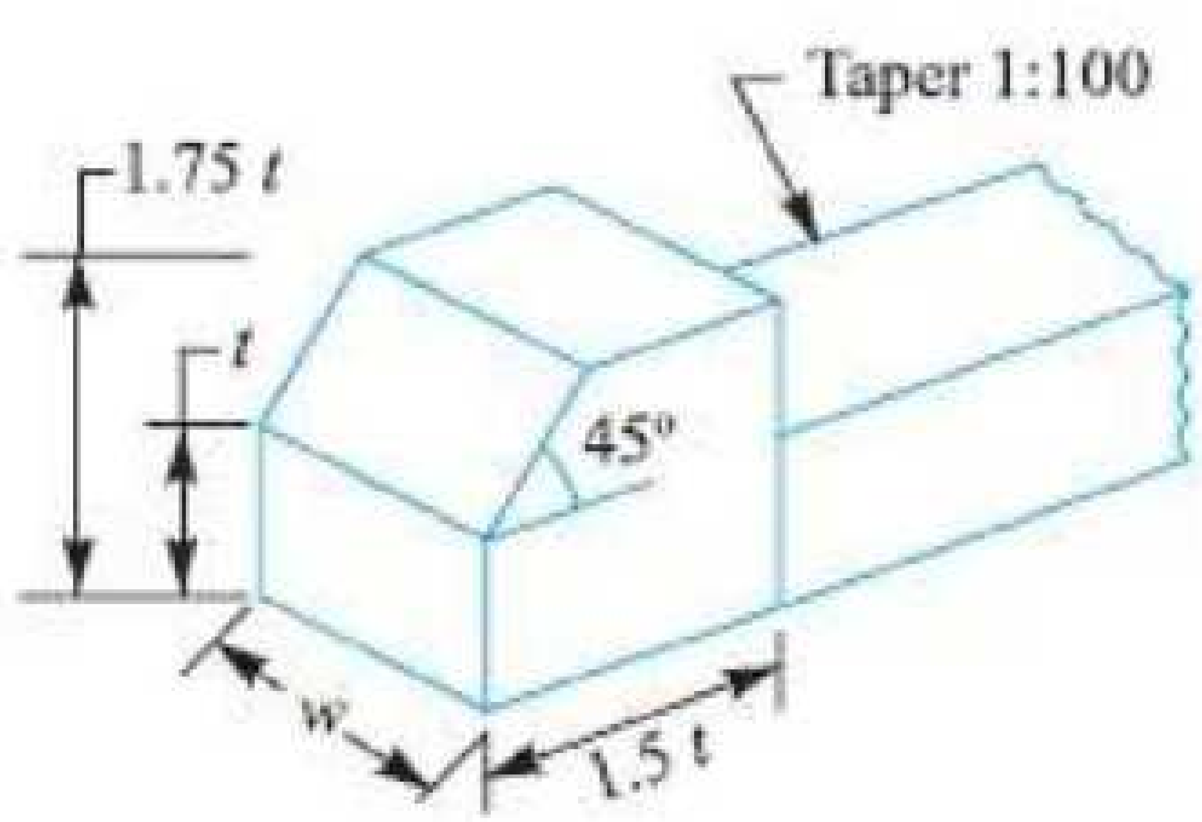
The key has taper 1 in 100 on the top side only.



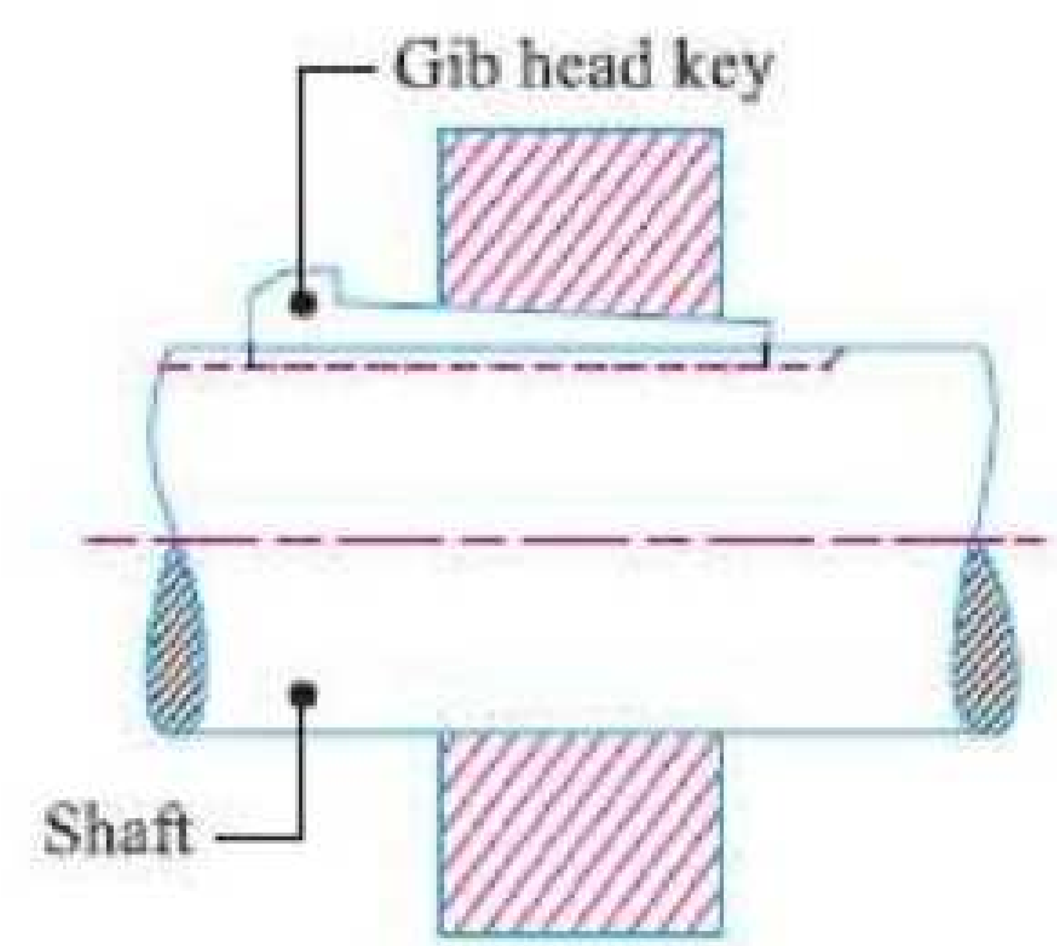
2. *Square sunk key.* *The only difference* between a rectangular sunk key and a square sunk key is that its width and thickness are equal, ***i.e.*** $w = t = d / 4$

3. *Parallel sunk key.* *The parallel sunk* keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. *Gib-head key.* *It is a rectangular sunk* key with a head at one end known as *gib head.* It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. (*a) and its use in shown in Fig. (b).*



(a)

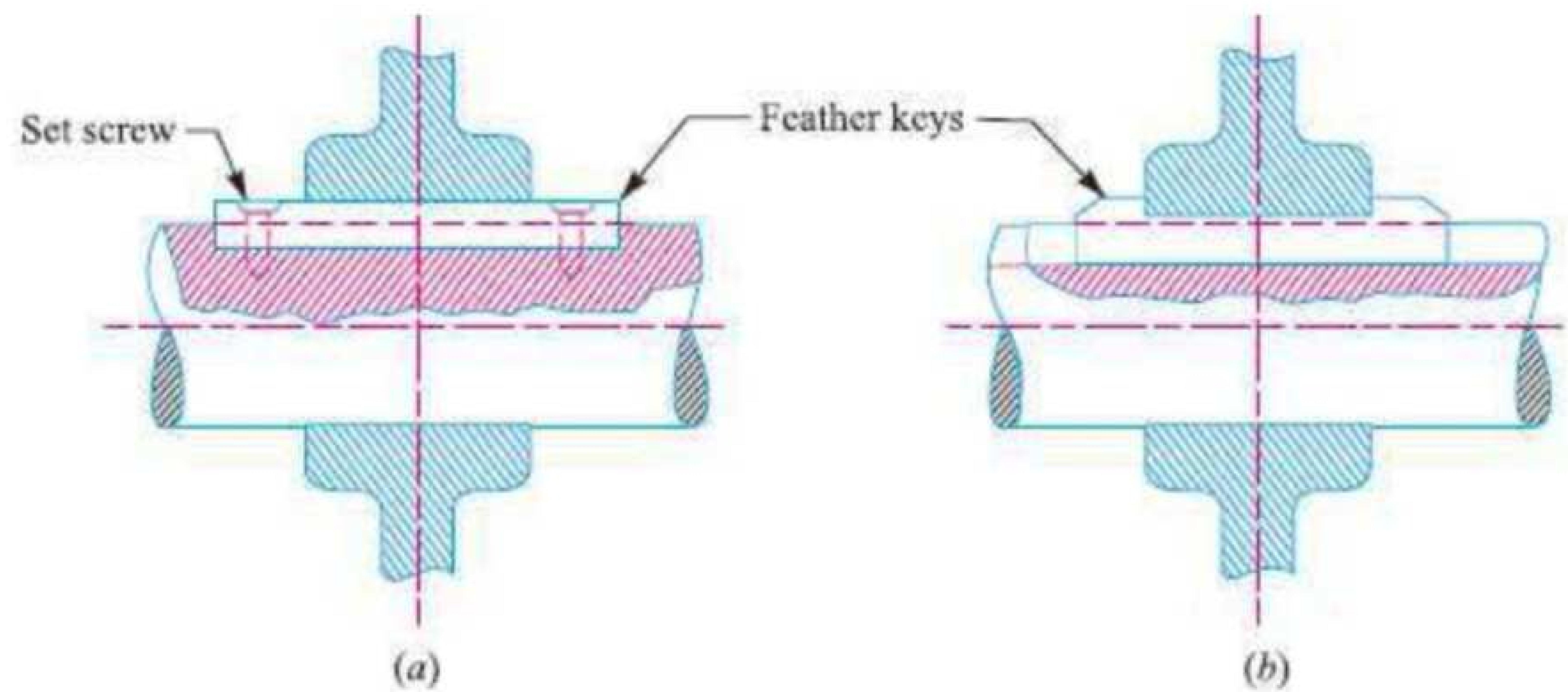


(b)

The usual proportions of the gib head key are :
Width, $w = d / 4$;and thickness at large end, $t = 2w / 3 = d / 6$

5. *Feather key.* *A key attached to one member of a pair and which permits relative axial* movement is known as *feather key. It is a special type of parallel key which transmits a turning* moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft as shown in Fig. (*a*) *or it may have double gib* heads as shown in Fig. (*b*). *The various proportions of a feather key are same as that of* rectangular sunk key and gib head key.



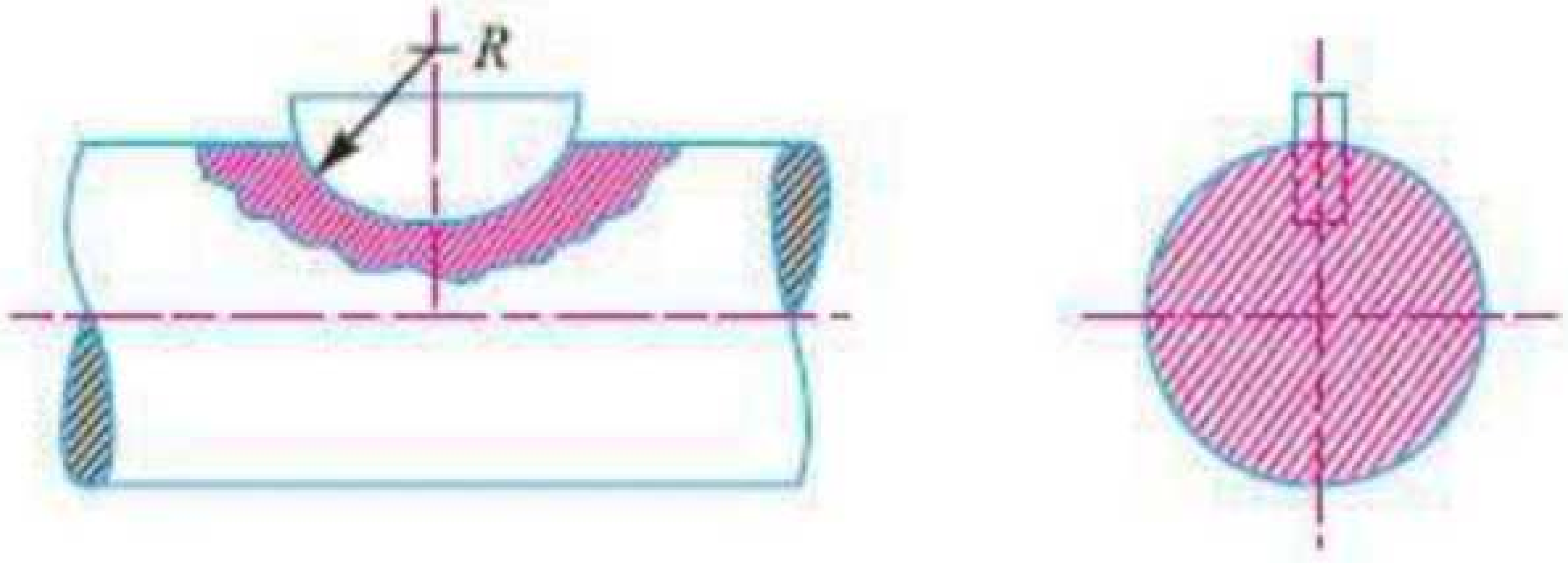
6. ***Woodruff key.*** *The woodruff key is an easily adjustable key. It is a piece from a cylindrical* disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

The main advantages of a woodruff key are as follows :

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.



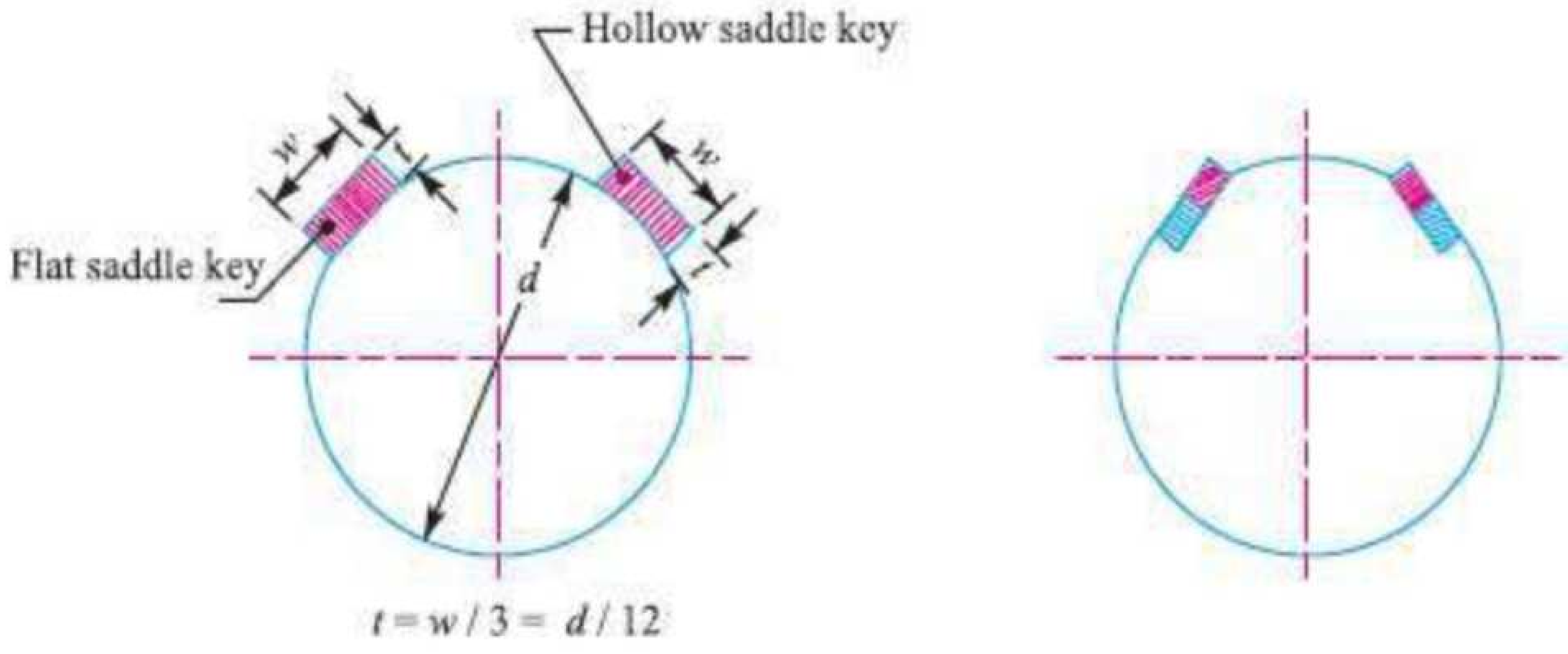
Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and 2. Hollow saddle key.

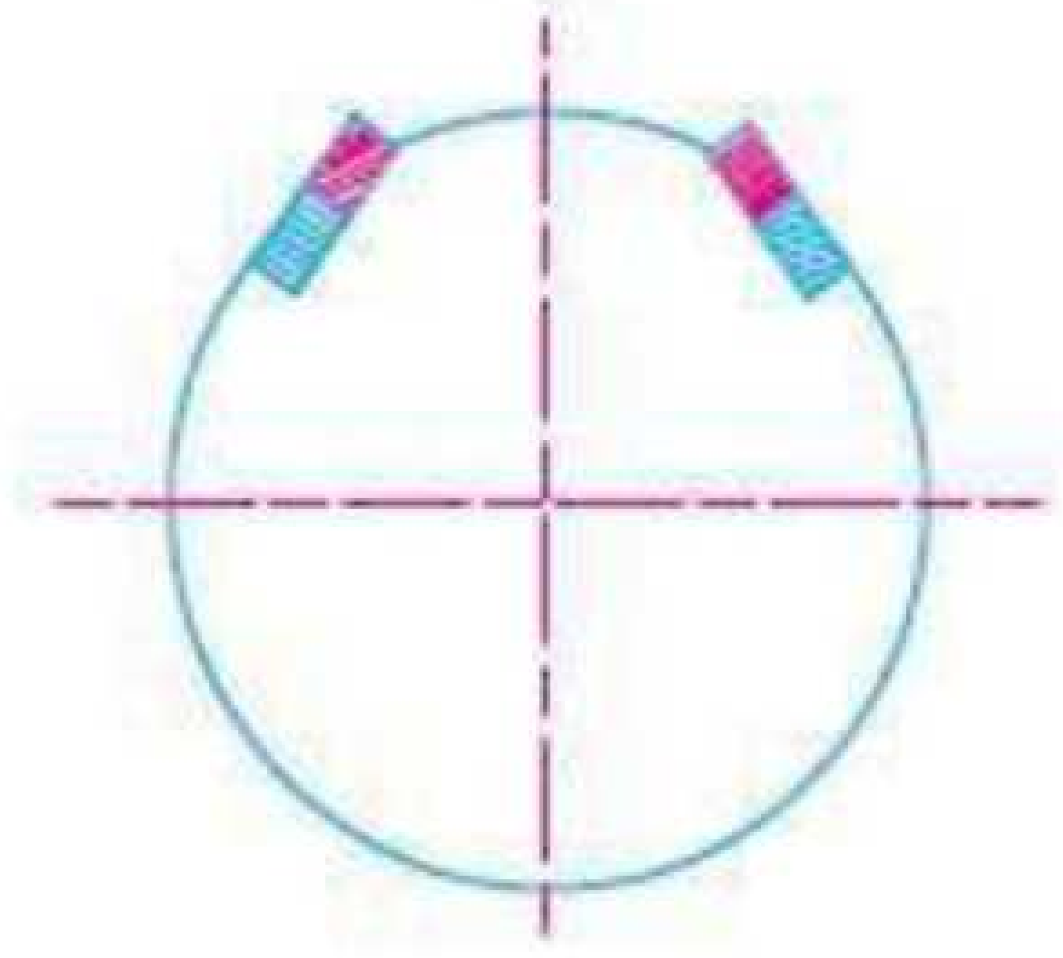
A ***flat saddle key*** *is a taper key which fits in a keyway in the hub and is flat on the shaft as shown* in Fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

A ***hollow saddle key*** *is a taper key which fits in a keyway in the hub and the bottom of the key* is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.



Tangent Keys

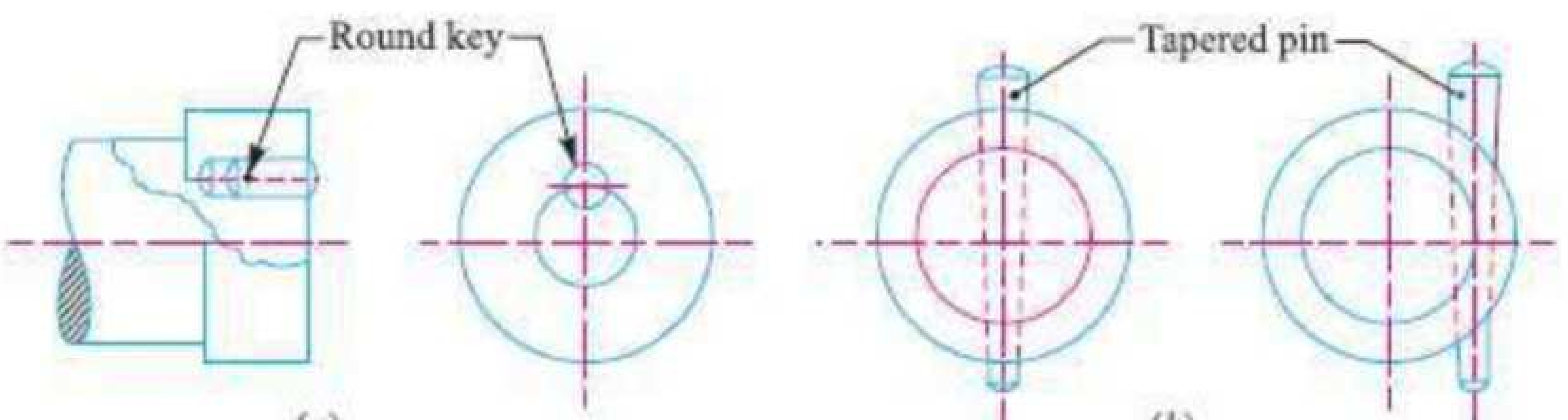
The tangent keys are fitted in pair at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



Round Keys

The round keys, as shown in Fig.(*a*), *are circular in section and fit into holes drilled partly* in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

Sometimes the tapered pin, as shown in Fig.(*b*), *is held in place by the friction between the* pin and the reamed tapered holes.



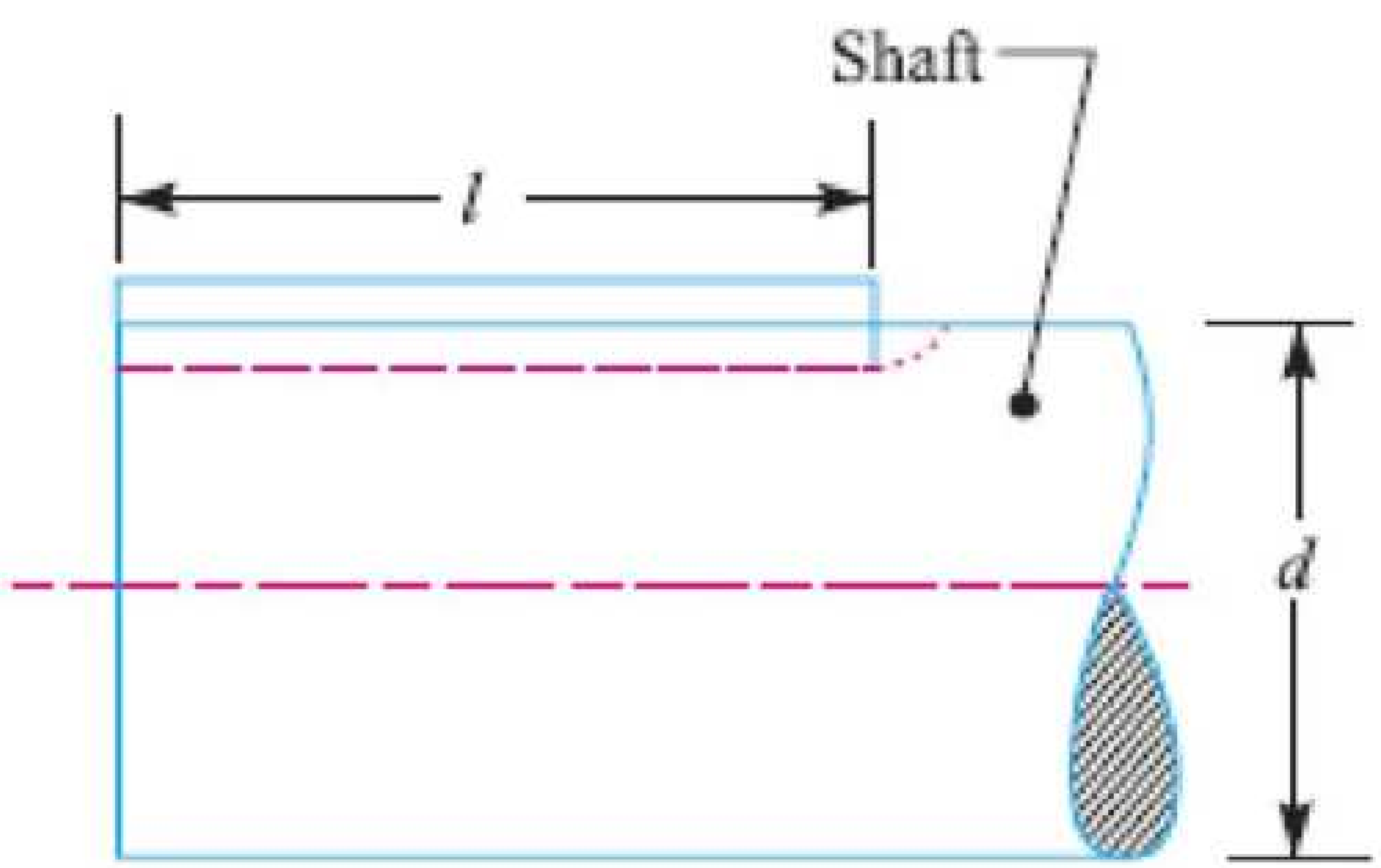
(*a*) (*b*)

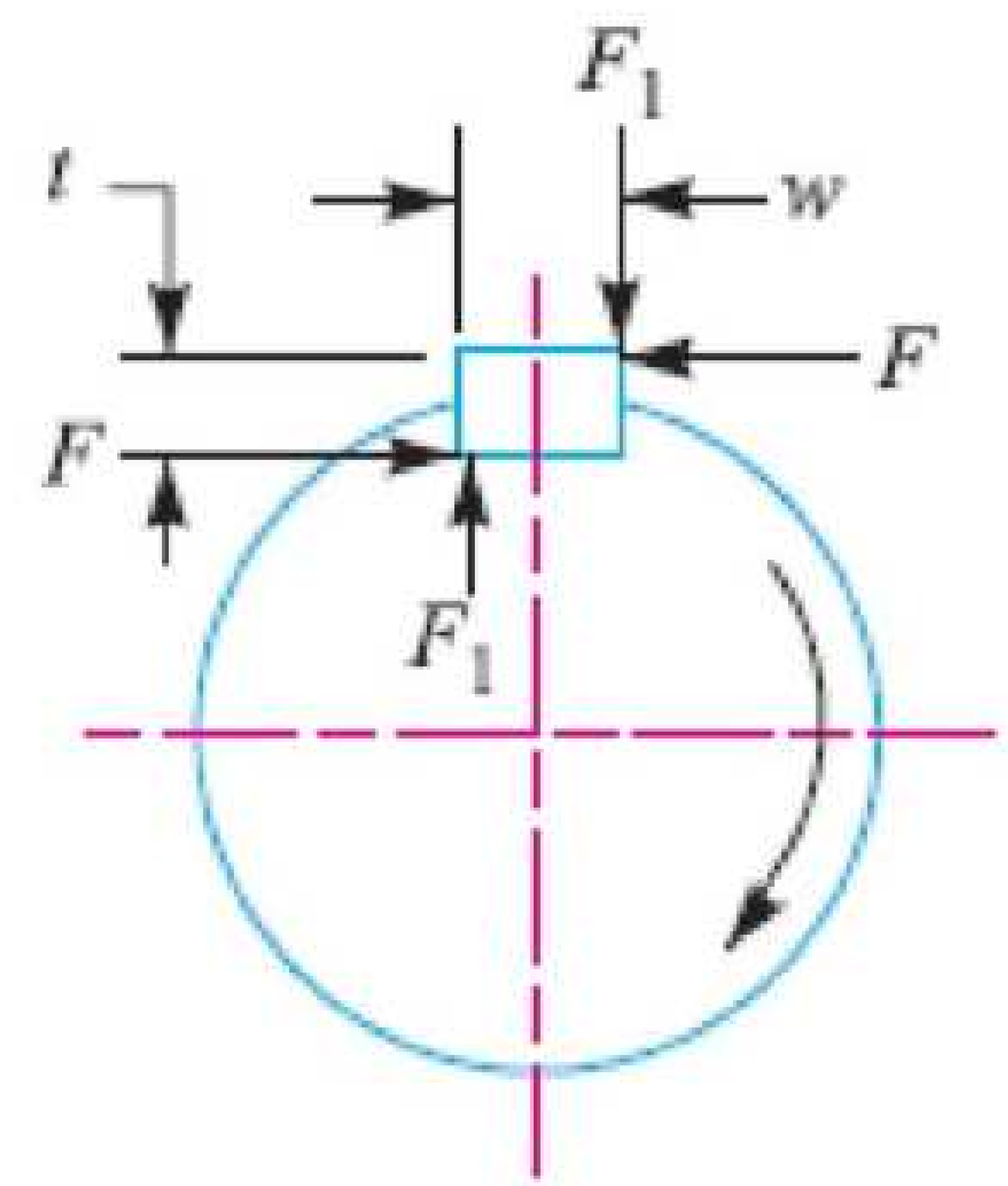
Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces ($F1$) *due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered* key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) *due to the torque transmitted by the shaft. These forces produce shearing and* compressive (or crushing) stresses in the key.





Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig.

Let T = *Torque transmitted by the shaft,*
F = *Tangential force acting at the circumference of the shaft,*
d = *Diameter of shaft,*
l = *Length of key,*
w = *Width of key.*
t = *Thickness of key, and*
τ and σc = *Shear and crushing stresses for the material of key.*

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...[\text{Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad ...(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation (*iii*), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{...}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad \text{...}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad \text{... (Taking } w = d/4) \quad \text{...}(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \qquad \text{... [From equation (vi)]}$$

Example 1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\ 800\ l \text{ N-mm} \quad \text{...}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\ (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\ 800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\ l \text{ N-mm} \quad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$l = 117.7$ say 120 mm **Ans.**

Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.

2. It should transmit the full power from one shaft to the other shaft without losses.

3. It should hold the shafts in perfect alignment.

4. It should reduce the transmission of shock loads from one shaft to another shaft.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

1. *Rigid coupling*. *It is used to connect two shafts which* are perfectly aligned. Following types of rigid coupling are important from the subject point of view :
(*a*) *Sleeve or muff coupling.*
(*b*) *Clamp or split-muff or compression coupling, and*
(*c*) *Flange coupling.*

2. *Flexible coupling*. *It is used to connect two shafts* having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :
(*a*) *Bushed pin type coupling,*
(*b*) *Universal coupling, and*
(*c*) *Oldham coupling.*

Sleeve or Muff-coupling

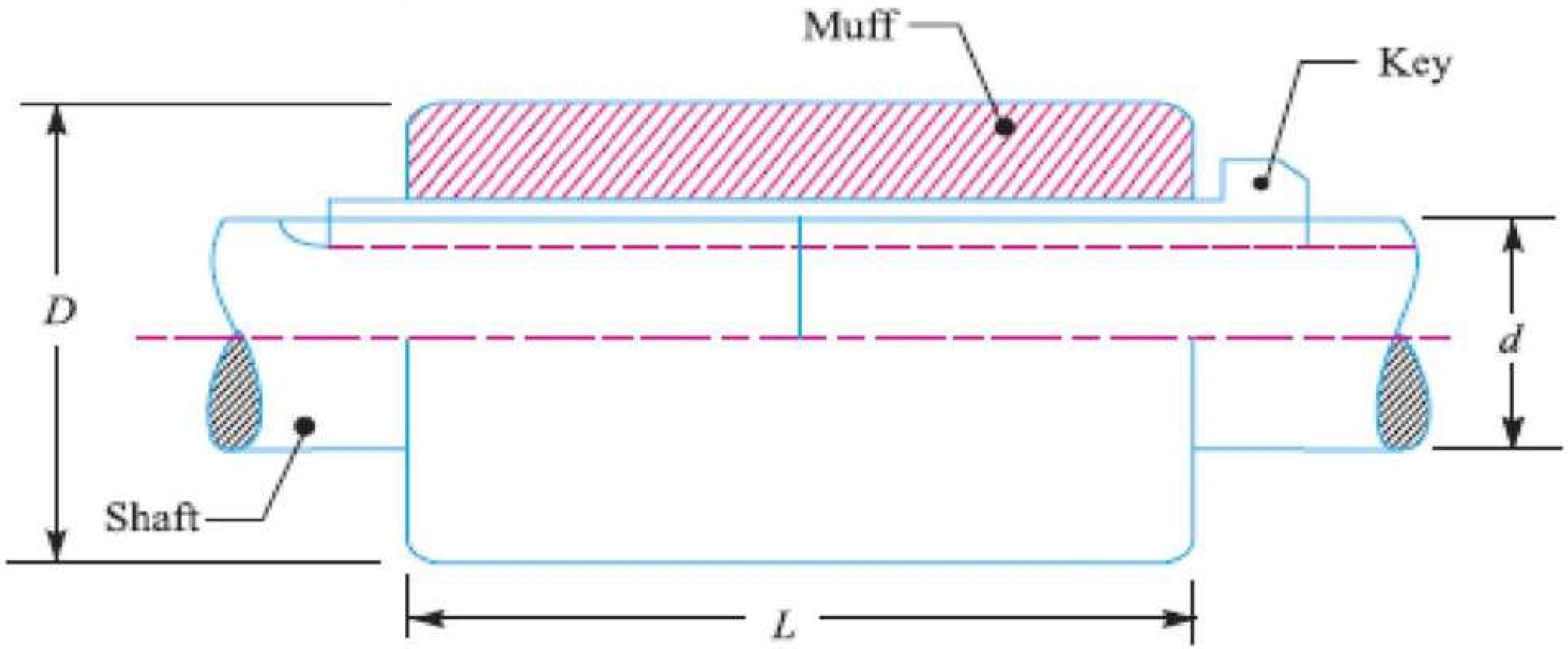
It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, $D = 2d + 13$ mm
and length of the sleeve, $L = 3.5\, d$

where *d is the diameter of the shaft.*
In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. *Design for sleeve*

The sleeve is designed by considering it as a hollow shaft.



Let T = *Torque to be transmitted by the coupling, and*

τ_c = *Permissible shear stress for the material of the sleeve which is cast iron.*

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. *Design for key*

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Example 1. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 Mpa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : *P = 40 kW = 40 × 103 W; N = 350 r.p.m.; τs = 40 MPa* = *40 N/mm2; σcs = 80 MPa* = 80 N/mm2; *τc = 15 MPa = 15 N/mm2*

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm} \quad \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

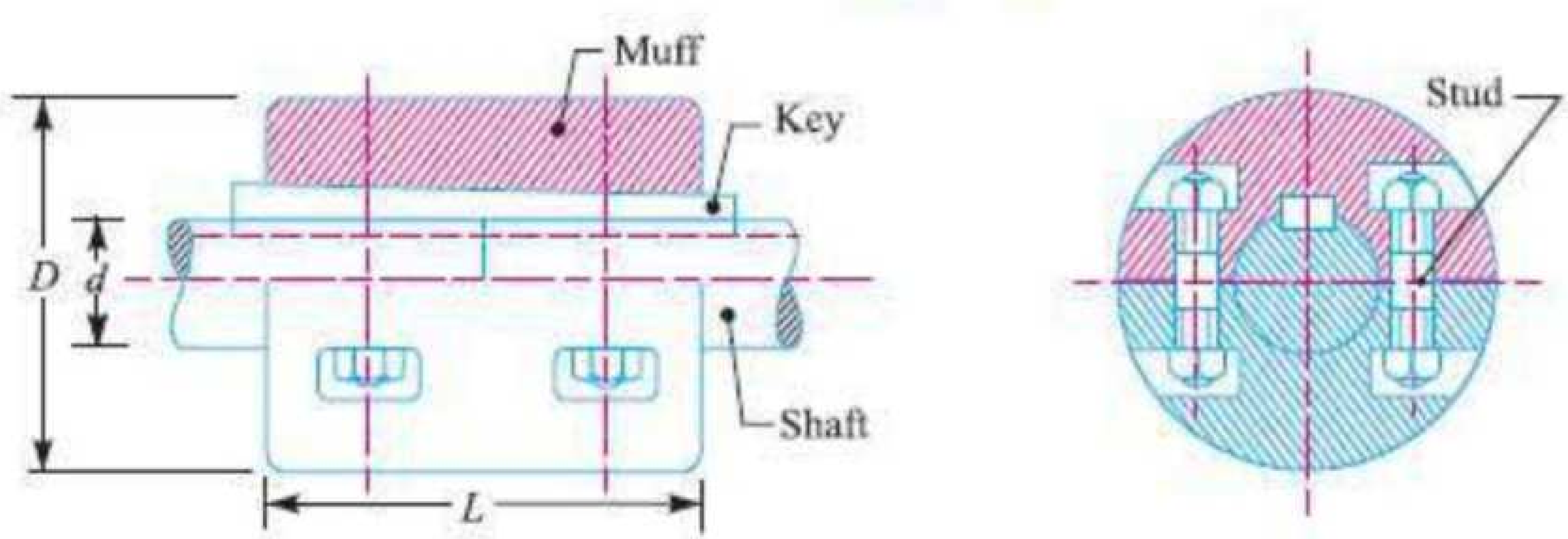
Clamp or Compression Coupling

It is also known as ***split muff coupling***. ***In*** this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are :

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\ d$

where d = Diameter of the shaft.



In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling.

2. *Design of clamping bolts*

Let T = *Torque transmitted by the shaft,*

d = *Diameter of shaft,*

d_b = *Root or effective diameter of bolt,*

n = *Number of bolts,*

σ_t = *Permissible tensile stress for bolt material,*

μ = Coefficient of friction between the muff and shaft,

L = *Length of muff.*

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

∴ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 1. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm}$$ **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm}$$ **Ans.**

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm}$$ **Ans.**

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22$ mm **Ans.**

Thickness of key, $t = 14$ mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\;70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	Key cross-section		Shaft diameter (mm) upto and including	Key cross-section	
	Width (mm)	Thickness (mm)		Width (mm)	Thickness (mm)
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50