unit-3

Mechanics

- It is the branch of physics which deals with the study of rest and motion. bodies.

Kinematics

- It is the branch of mechanic which deals with the study of rest & motion with out going into the cause of motion.

Rest and motion

- A body is said to be at rest if it does not change its position with respect to its surrounding.
- A body is said to be at motion if it changes its position with respect to its surrounding.

Absolute - Rest and motion
(zero)

- Rest and motion said to be absolute if the reference point is at absolute rest.
- Since there is no reference point in the universe ~~point~~ is at absolute rest, absolute rest & motion is impossible. It is just a theoretical concept.
- Hence rest and motion is relative

Distance (s)..

- The path covered by a body when moving from one point to another is called distance (s).
- It is a scalar quantities.

Displacement ($\vec{s}$)

Displacement of a body is define as the shortest distance between initial and final point and directed towards the final point.

- It is a vector quantity and denoted by 's'.
- Displacement may be positive, negative or zero. Sence It is a vector quantity.

Speed

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

- Given distance = 560
  Time = 80

$$\frac{560 \times 1000}{3600 \times 7} = \frac{200}{9} = 22.23\, m/s$$

- It is defined as the rate of change of distance covered by a body.

i.e $\boxed{v = \frac{s}{t}}$

- It is a scalar quantity.

Unit

In CGS

$$v = \frac{cm}{sec} = cm/s = cms^{-1}$$

In MKS

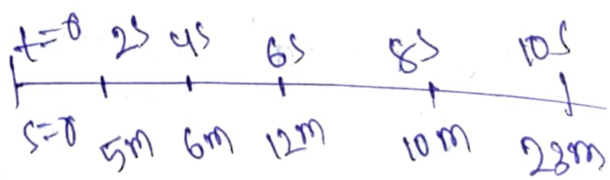
$$v = \frac{m}{sec} = m/s = ms^{-1}$$

- The biger unit km/hr is also use.
- When a body is in a non-uniform speed.

The instantaneous speed

$$\boxed{v = \frac{ds}{dt}}$$

(where $\Delta t \to 0$)



Velocity ($\vec{v}$)

$$\frac{\text{Displacement}}{\text{time}} = \frac{\vec{s}}{t}$$

- It is defined as the rate of change of displacement of a body.

i.e $\boxed{\vec{v} = \frac{\vec{s}}{t}}$

- It is a vector quantity.

unit

In CGS = cm/s

SI = m/s

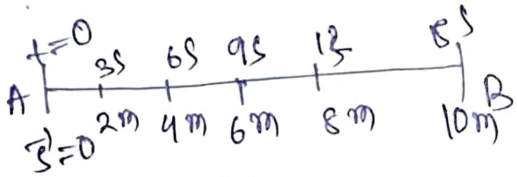
For instantaneous velocity.

$\Delta t \to 0$

$$\boxed{\vec{v} = \frac{\Delta s}{\Delta t} = \frac{d\vec{s}}{dt}}$$

Uniform velocity (Constant velocity)

- A body is said to be move in uniform velocity if it covers equal displacement in equal interval of time.

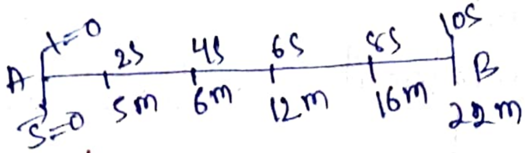


$t = 3s$
$s = 2m$ $\quad \vec{v} = \frac{\vec{s}}{t} = \frac{2}{3} = 0.667\,m/s$

0.66

Non-uniform velocity (variable velocity)

- A body is said to be move in non-uniform velocity if it covers un equal displacement in equal interval of time.



- Accelaration ($\vec{a}$)
- It is defind as the rate of change of velocity of a body.

$$\boxed{\vec{a} = \frac{\Delta \vec{v}}{\Delta t}}$$

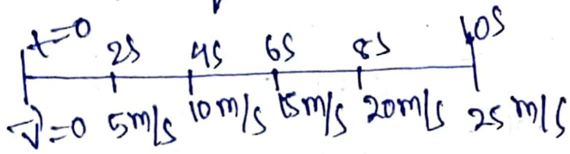
For instantaneous accelaration $\Delta t \to 0$

$$\boxed{\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}}$$

- There are two types of accelaration.

① Uniform-accelaration (Constant accelaration)

- A body is said to be a constant accelaration if its velocity changes equal amounts in equal interval of time.



$$\vec{a} = \frac{\vec{v}}{t} = \frac{5}{2} = 2.5\,m/s^2$$

(+ve uniform accelaration)

② Non-uniform accelaration (Variable accelaration)

- A body is said to be move on non-uniform accelaration if velocity changes in equal amount in equal interval of time.

Expression of accelaration interms of displacement

for instantaneous accelaration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right)$$

$$= \frac{d^2\vec{s}}{dt^2}$$

$$\vec{a} = \frac{d^2\vec{s}}{dt^2}$$

Integration

By integration we can calculate the entire value of a quantity, when a small value of qty is known.

* $\int dx = x$
* $\int x^n dx = \frac{x^{n+1}}{n+1}$
* $\int_u^v dx = [x]_u^v = v - u$

Q: $\int 2dv = 2\int dv = 2v$

Q: $\int_0^t \frac{1}{2} dt = \frac{1}{2}\int_0^t dt$

$$= \frac{1}{2}[t]_0^t$$

$$= \frac{1}{2}(t-0)$$

$$= \frac{1}{2}t \text{ or } \frac{t}{2}$$

Q: $\int_0^t t^2 dt = \int_0^t \frac{t^{2+1}}{2+1} dt$

$= \int_0^t \frac{t^3}{3} dt$

$= \left[\frac{t^3}{3}\right]_0^t$

$= \frac{t^3}{3} - 0$

$= \frac{t^3}{3}$

Q: $\int_u^v \frac{1}{3} v\, dv = \frac{1}{3}\int_u^v v\, dv$

$= \frac{1}{3}\int_u^v \frac{v^{1+1}}{1+1} dv$

$= \frac{1}{3}\int_u^v \frac{v^2}{2} dv$

$= \frac{1}{3}\left[\frac{v^2}{2}\right]_u^v$

$= \frac{1}{3}\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$

$= \frac{1}{3} \times \frac{1}{2}(v^2 - u^2)$

$= \frac{1}{6}(v^2 - u^2)$

Q: $\int_0^s \frac{1}{2s^3} ds = \frac{1}{2}\int_0^s \frac{1}{s^3} ds$

$= \frac{1}{2}\int_0^s s^{-3} ds$

$= \frac{1}{2}\int_0^s \frac{s^{-3+1}}{-3+1} ds$

$= \frac{1}{2}\int_0^s \frac{s^{-2}}{-2} ds$

$= \frac{1}{2}\left[\frac{s^{-2}}{-2}\right]_0^s$

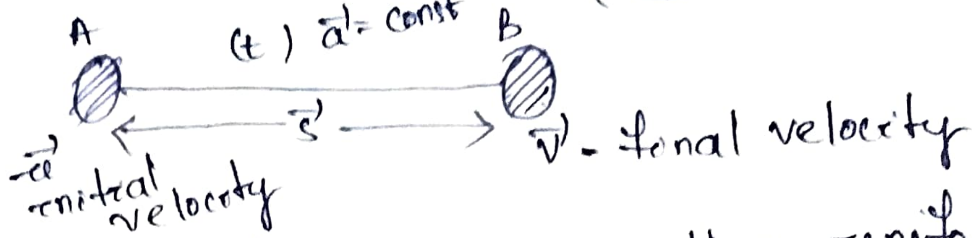
$= \frac{1}{2}\left[\frac{s^{-2}}{-2} - 0\right)$

$= \frac{1}{2} \cdot \frac{1}{-2} (s^{-2})$

$= \frac{1}{-4} s^{-2}$

Equation of motion (in one dimension)

(Calculus method)



- Consider a body moving with a uniform accelaration '$\vec{a}$' passing a point 'A' with an initial velocity '$\vec{u}$' and reaches to a point 'B' with final velocity '$\vec{v}$' after covering a distance '$\vec{s}$' let 't' is the time taken by the body.
- The equation is involving $\vec{u}, \vec{v}, \vec{s}, \vec{a}$ & t are called equation of motion.

Velocity after time 't' ($v = u + at$)

* For instantaneous accelaration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$d\vec{v} = \vec{a}\, dt$$

integrating both sides

$$\int_u^v dv = \int_0^t a\, dt$$

$$\Rightarrow [v]_u^v = a[t]_0^t$$

$$\Rightarrow v - u = at$$

$$\Rightarrow v = u + at$$

(first equation of motion)

- Distance Covered in time 't'

$(\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2)$

- The instantaneous velocity of a body is given by.

$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$d\vec{s} = \vec{v}\,dt$$

$$\Rightarrow d\vec{s} = (\vec{u} + \vec{a}t)\,dt$$

$$\Rightarrow \int_0^s d\vec{s} = \int_0^t \vec{u}\,dt + \int_0^t \vec{a}t\,dt$$

$$\Rightarrow [s]_0^s = \vec{u}[t]_0^t + \vec{a}\left[\frac{t^2}{2}\right]_0^t$$

$$\Rightarrow (s - 0) = \vec{u}(t - 0) + \vec{a}\left(\frac{t^2}{2} - 0\right)$$

$$\Rightarrow s = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

(2nd equation of motion,)

velocity after Covering a distance 's'

$$v^2 - u^2 = 2as$$

$$\Rightarrow v^2 = u^2 + 2as$$

Inst. acc^n of a body is given by

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d\vec{v}}{d\vec{s}} \cdot \frac{d\vec{s}}{dt}$$

$$= \frac{d\vec{v}}{d\vec{s}} \cdot \vec{v}$$

$$\vec{a}\,d\vec{s} = \vec{v}\,d\vec{v}$$

Integrating both sides.

$$\int_{u}^{v} \vec{v}\,dv = \int_{0}^{s} \vec{a}\,d\vec{s}$$

$$\Rightarrow \left[\frac{v^2}{2}\right]_u^v = \vec{a}\,[s]_0^s$$

$$\Rightarrow \left[\frac{v^2}{2} - \frac{u^2}{2}\right] = \vec{a}(\vec{s} - 0)$$

$$\Rightarrow \frac{1}{2}(v^2 - u^2) = \vec{a}\vec{s}$$

$$\Rightarrow v^2 - u^2 = 2as$$

$$\Rightarrow v^2 = a^2 + 2as$$

Distance Covered in nth Second

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$d\vec{s} = \vec{v}\,dt$$

$$\int_{s_{n-1}}^{s_n} d\vec{s} = \int_{n-1}^{n} (\vec{u} + \vec{a}t)\,dt$$

$$\Rightarrow [s]_{s_{n-1}}^{s_n} = \int_{n-1}^{n} u\,dt + \int_{n-1}^{n} at\,dt$$

$$\Rightarrow S_n - S_{n-1} = u[t]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

$$S_{nth} = u[n-(n-1)] + a\left[\frac{n^2}{2} - \frac{(n-1)^2}{2}\right]$$

$$S_{nth} = u(\cancel{n}-\cancel{n}+1) + \frac{a}{2}\left[n^2 - [n^2+1^2-2n]\right]$$

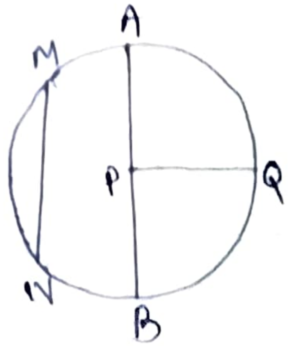
$$S_{nth} = u + \frac{a}{2}[n^2 - n^2 - 1 + 2n]$$

$$\text{or} \quad \boxed{S_{nth} = u + \frac{a}{2}(2n-1)}$$

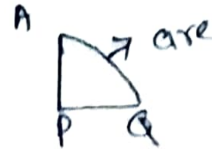
body is dropped From a height 'h'

$S_{nth} = h_{nth}$, $u = 0$, $a = g$

$$\boxed{h_{nth} = \frac{g}{2}(2n-1)}$$



P = Centre
$\overline{PQ}$ = Radius
AB = diameter
MN = chord (জ্যা)



Circular motion

- A body is said to be move in circular motion if its distance from a fixed point allways remains constant the fixed point is called centre and the fixed distance is called radius and the path is a circle.

Angular distance

- It is defined as the angle turned by the radius vector in circular motion.

$$\text{angle } \theta = \frac{\text{length of the arc}}{\text{radius}}$$

Angular displacement

$$\theta = \frac{\text{length of the arc}}{\text{radius}}$$

$$\theta = \frac{\vec{S}}{r}$$

unit of θ = radian

$$\vec{S} = r\theta$$

length of the arc = radius x Angular displacement

Angular velocity ($\vec{\omega}$) - small omega

- It is defined as the rate of change of angular displacement of the body.

i.e $\boxed{\vec{\omega} = \frac{\vec{\theta}}{t}}$

unit of $\vec{\omega} = \frac{\text{radian}}{\text{second}} = \text{rad/sec}$

Relation between $\vec{\omega}$ & $\vec{v}$, linear velocity

$$\vec{\omega} = \frac{\theta}{t} = \frac{\vec{s}/r}{t}$$

$$= \frac{1}{r} \cdot \frac{\vec{s}}{t}$$

$$= \frac{\vec{v}}{r}$$

$$\vec{\omega} = \frac{\vec{v}}{r}$$

$$\boxed{\vec{v} = \vec{\omega} r}$$

Angular accelaration (α)

- It is defined as the rate of change of angular velocity of the body.

i.e $\boxed{\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}}$

unit $= \frac{\text{rad/s}}{s} = \text{rad/sec}^2$

Relation between $\vec{\alpha}$ & $\vec{a}$

$$\alpha = \frac{\Delta \vec{\omega}}{\Delta t} = \frac{\Delta \vec{v}/r}{\Delta t}$$

$$= \frac{1}{r} \cdot \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{1}{r} \cdot \vec{a}$$

$$\boxed{\vec{\alpha} = \frac{\vec{a}}{r}}$$

or $\boxed{\vec{a} = r\vec{\alpha}}$

Linear accelaration = radius x angular accelaration

Time period (T)

- It is defined as the time taken by the body to complete one rotation.

$$\omega = \frac{\theta}{t}$$

$$\boxed{\omega = \frac{2\pi}{T}} \qquad \pi = \frac{22}{7}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

Frequency (f)

- It is defined as the number of rotation may by the particle in one second.

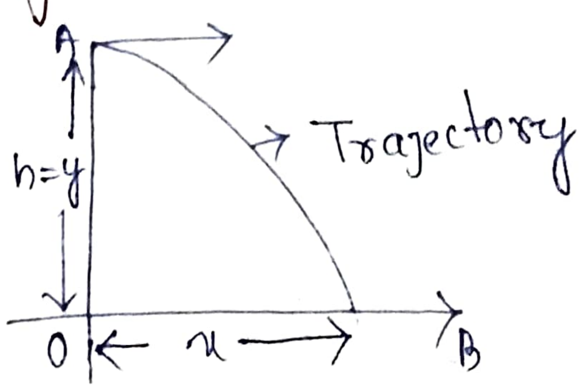
$$\boxed{f = \frac{1}{T}}$$

projectile

- It is defined as a body which is thrown or project into the space a moves freely under gravity and fall the ground.

Ex ① The cricket ball thrown into the spa
② A bullet is fired from a gun.
③ A bag is droped from a moving aeroplane.

projectile fired horizontally at a height from ground (Horizontal projectile):-



- Consider a projectile is projected with velocity $\vec{u}$ horizontally.
  h is the height above the ground.
- Horizontally velocity $\vec{u}$ is uniform throughout its path because there is no gravity in horizontal direction ($\vec{a} = 0$)
- vertical velocity is non-uniform, it increases due to gravity.
  - Horizontal distance covered equal to x.
  - vertical distance covered equal to y.
- path covered due to projectile motion is called trajectory.

Equation of trajectory

Applying the equation

$$S = ut + \frac{1}{2}at^2$$

① For horizontal motion (x)

$$x = ut + 0 \quad (\because \vec{a} = 0)$$

$$x = ut$$

$$\boxed{t = \frac{x}{u}} \text{ —— } ①$$

vertical motion (y)

$$S = ut + \frac{1}{2}at^2$$

$$y = 0 + \frac{1}{2}at^2$$

$$\boxed{y = \frac{at^2}{2}} \text{ —— } ②$$

$$y = \frac{a}{2}\left(\frac{x^2}{u^2}\right)$$

$$y = \frac{a}{2u^2}x^2 \quad \left(\because k = \frac{a}{2u^2}\right)$$

$\Rightarrow \boxed{y = kx^2}$ —— (3) (parabolic eq^n)

Time period (t)

$$S = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}at^2$$

$$h = \frac{1}{2}gt^2$$

$$h = \frac{gt^2}{2}$$

$$\frac{gt^2}{2} = h$$

$$t^2 = \frac{2h}{g}$$

$$\boxed{t = \pm\sqrt{\frac{2h}{g}}}$$ —— (4)

Horizontal range

$$S = ut + \frac{1}{2}at^2$$

- it is the horizontal distance covered by the projectile.

$$x = ut + 0$$

$$x = ut$$

putting the value of t

$$\boxed{x = u\sqrt{\frac{2h}{g}}}$$ —— (5)