STRUCTURAL MECHANICS

Basic concepts Chapter- 1

Mechanics:- It is the branch of science, which deals with the study of force and effect of force on body. Mechanics may be devided into 3 parts

i) Engineering Mechanics

ii) Structural Mechanics

iii) Fluid Mechanics

Force:- It is an external agent which causes to destroy the state of a body.

Inertia:- Inability to move / change of an object.
* we can calculate value of inertia.

Moment of inertia:-

The turning effect produced by an object.

or

Inability to move incase of rotational / circular motion.

Mechanics:-

For Rigid body → Engineering Mechanics

For deformable body → Structural Mechanics

For Fluid → Fluid Mechanics

Rigid body	Deformable body
→ $F_{net} \neq 0$ (net force)	→ $F_{net} = 0$
→ Displacement	→ Deformation
→ Distance between two point does not change.	→ Distance between two point change.

~~Engineering Mechanics~~

Engineering Mechanics

It is the study of mechanics of Rigid body.

Rigid body:-

It may be defined as an idealization of solid body where net force is acting on the body is not equal to zero. ($\neq 0$)

In others word, the distance between any two points on the body does not change, irrespective application of force/ load i.e. displacement occur in the body.

Structural Mechanics

It is the study of mechanics of deformable body.

Deformable body:-

It is the body where net force acting on the body is ~~zero~~ equal to zero. ($= 0$)

In other words, the distance between any two points on the body changes after the applications of force / load, i.e. deformation occurs.

Fluid Mechanics

It is the study of mechanics of fluid.

Fluid:-

It is the any substances that has the ability to flow.

Moment:-

The turning effect produced by ~~on~~ a body is called moment.

$$\boxed{M = F \times \text{perpendicular distance}}$$

S.I unit = Nm

Couple:-

- When two equal and opposite forces acting on a body at ~~on~~ a distance then, the effect of moment of force is known as couple.

$$\boxed{\text{Couple} = F \times d}$$ SI unit = Nm.

Torque:-

- When the moment is applied on along longitudinal axis of the member is called torque.

S.I unit = Nm

Conditions of Equilibrium:-

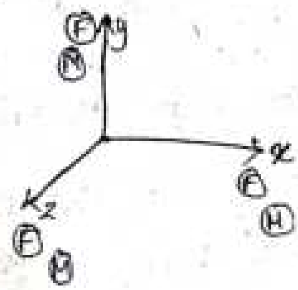
- It may be defined as a state of rest or uniform motion of a body, ~~where~~ whose net force and net moment is acting upon a body is equal to zero.

i.e.

$\Sigma f_x = 0$ $\Sigma M_x = 0$

$\Sigma f_y = 0$ $\Sigma M_y = 0$

$\Sigma f_z = 0$ $\Sigma M_z = 0$



Support condition:-

Support conditions are mainly up off 3 types

i) Hinged support

ii) Roller support

iii) Fixed support

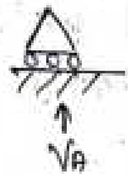
Hinged support:-

The supports which do not have any translational movement ,. only rotational movement is possible. These of support are known as hinged support.

It is denoted by  and it has two rea^n

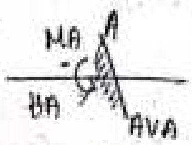
2) Roller support:-

The support in which translational moment is possible in (x) direction only and rotational movement is possible these type of support are roller support.

It is denoted by  and it has 1 rean.

3) Fixed support:-

The support in which neither translational nor rotational movement is possible, these type of support are known as fixed support.

It is denoted by  and it has 3 rean.

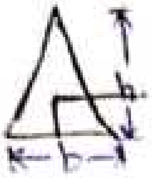
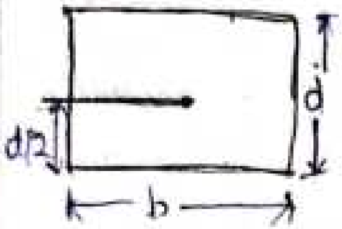
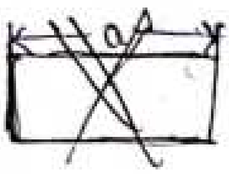
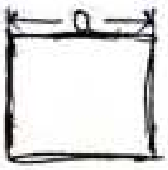
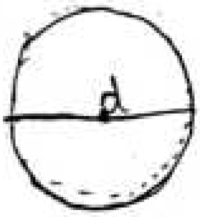
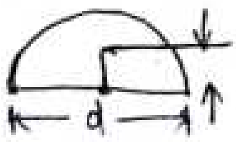
Free body diagram:-

The schematic representation of forces and reactions on a body is known as free body diagram.

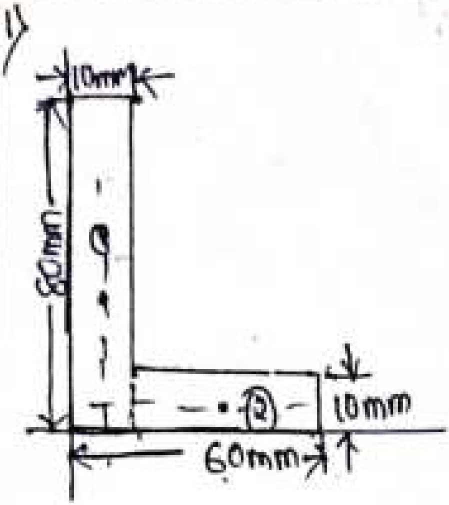
C.G and MI of different sections:-

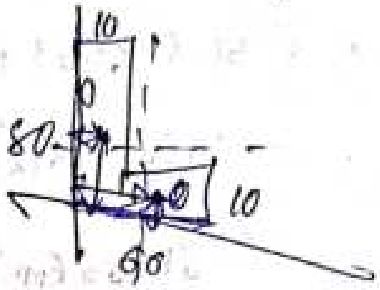
Shape	section	CG	MI

CG and MI of different sections:-

Shape	Section	CG	MI
 $h/3$	Triangle	$h/3$	$\frac{bh^3}{36}$
	Rectangle	$d/2$	$I_{xx} = bd^3/12$ $I_{yy} = db^3/12$
			
	Square	$a/2$	$I_{xx} = I_{yy}$ $= a^4/12$
	circle	$d/2$	$\frac{\pi d^4}{64}$
 $4r/3\pi$	semicircle	$\frac{4r}{3\pi}$	$\frac{\pi d^4}{128}$

1)





$a_1 = 80 \times 10 = 800 \text{ mm}^2$

$a_2 = (60 - 10) \times 10 = 500 \text{mm}^2$

$x_1 = 5 \text{ mm}$

$x_2 = 30\text{mm}$

$I_{xx_1} = \frac{bd^3}{12} + A_1 h_1^2$

$y_1 = 40\text{mm}$

$y_2 = 5 \text{ mm}$

$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(800 \times 5) + (500 \times 30)}{1300} = 14.61\text{mm}$

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(800 \times 40) + (500 \times 5)}{1300} = 26.53 \text{ mm}$

$I_{xx} = I_{xx_1} + I_{xx_2}$

$I_{xx_1} = 10 \times \frac{80^3}{12} + 800 \times (13.47)^2$ $(40 - 26.53)$

$= 10 \times 42666.6 + 800 \times 181.44$

$= 426666 + 145152$

$= 571818 \text{ mm}^4$

$$I_{xx_2} = 50 \times \frac{10^3}{12} + 500 \times \cancel{35.3}\ (21.53)^2 \rightarrow (26.53 - 5)$$

$$= 50 \times 83.33 + 500 \times 463.54$$

$$= 4166.5 + 231770$$

$$= 235936.5 \text{ mm}^4$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= 571818 + 235936.5$$

$$= 807754.6 \text{ mm}^4$$

$$I_{yy_1} = 80 \times \frac{10^3}{12} + 800 \times (9.61)^2 \rightarrow (14.61 - 5)$$

$$= 80 \times 83.33 + 800 \times 92.35$$

$$= 6666.4 + 73880$$

$$= 80546.4 \text{ mm}^4$$

$$I_{yy_2} = 10 \times \frac{50^3}{12} + 500 \times (15.39)^2 \rightarrow (30 - 14.61)$$

$$= 10 \times 10416.66 + 500 \times 236.85$$

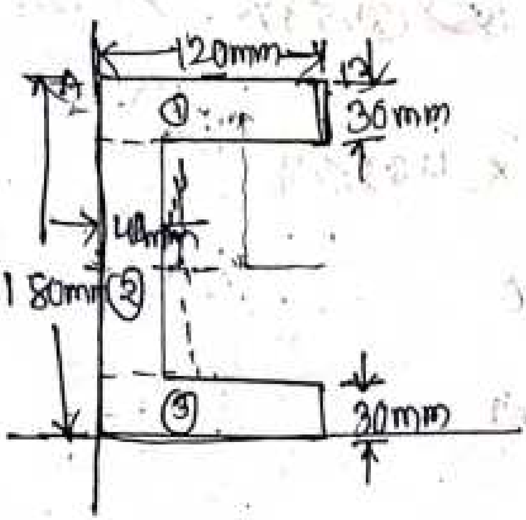
$$= 104166.6 + 118425$$

$$= 222591.6 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= 80546.4 + 222591.6 = 303138 \text{ mm}^4$$

2)



$a_1 = 120 \times 30 = 3600\ mm^2$

$a_2 = 120 \times 40 = 4800\ mm^2$

$a_3 = 120 \times 30 = 3600\ mm^2$

$x_1 = 60\ mm$, $y_1 = 165\ mm$

$x_2 = 20\ mm$, $y_2 = 90\ mm$

$x_3 = 60\ mm$, $y_3 = 15\ mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(3600 \times 60) + (4800 \times 20) + (3600 \times 60)}{(3600 + 4800 + 3600)}$$

$$= \frac{216000 + 96000 + 216000}{12000}$$

$$= \frac{528000}{12000}$$

$= 44\ mm$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{3600 \times 165 + 4800 \times 90 + 3600 \times 15}{12000}$$

$$= \frac{594000 + 432000 + 54000}{12000}$$

$$= \frac{1080000}{12000}$$

$$= 90\,mm$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{xx_1} = 120 \times \frac{30^3}{12} + 3600 \times (90 - 15)^2$$

$$= 120 \times 2250 + 3600 \times 5625$$

$$= 270000 + 20250000$$

$$= 20520000\,mm^4$$

$$I_{xx_2} = 40 \times \frac{120^3}{12} + 4800 \times (0)^2$$

$$= 40 \times 144000 + 0$$

$$= 5760000\,mm^4$$

$$I_{xx_3} = 120 \times \frac{30^3}{12} + 3600 \times (90-15)^2$$

$$= 120 \times 2250 + 3600 \times 5625$$

$$= 270000 + 20250000$$

$$= 20520000 \text{ mm}^4$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= 20520000 + 5760000 + 20520000$$

$$= 46800000 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$I_{yy_1} = 30 \times \frac{120^3}{12} + 3600 \times (60-44)^2$$

$$= 30 \times 144000 + 3600 \times 256$$

$$= 4320000 + 921600$$

$$= 5241600 \text{ mm}^4$$

$$I_{yy_2} = 120 \times \frac{40^3}{12} + 4800 \times (44-20)^2$$

$$= 120 \times 5333.3 + \cancel{576}\ 4800 \times 576$$

= ~~[illegible]~~

$= 640000 + 2764800$

$= 3404800 \text{ mm}^4$

$$I_{yy_3} = 30 \times \frac{120^3}{12} + 3600 \times (60-44)^2$$

$= 30 \times 144000 + 3600 \times 256$

$= 4320000 + 921600$

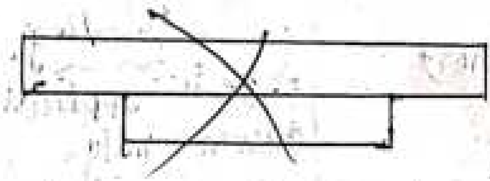
$= 5241600 \text{ mm}^4$

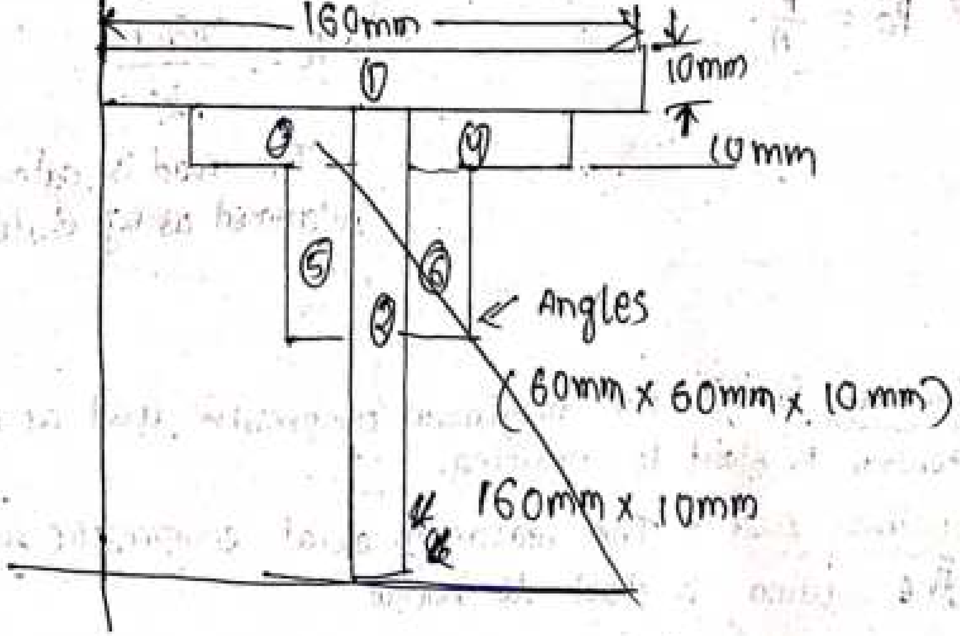
$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$= 5241600 + 3404800 + 5241600$

$= 13888000 \text{ mm}^4$

3)





Column and Struct

Column:- If the ~~struct~~ strut is vertical i.e. at 90° to the horizontal is known as column.

Strut:- A mebe on structure or bar which carries an axial compressive load is known as strut.

All columns are strut, but all strut are not columns.

Euler's Formula:-

$$\boxed{P_{euler} = \frac{\pi^2 EI}{le^2}}$$

where,

E = Young's modulus of elasticity

I = Moment of inertia

le = Effective length

Difference between crushing and Buckling:-

Crushing	Buckling
⇒ It is applicable to short column only	⇒ It is applicable to long column only.
⇒ $P_c = \frac{F}{A}$	⇒ $P_E = \frac{\pi^2 EI}{le^2}$ This load is ~~calculating~~ calculated using Euler's formula

Crushing load ⇒ The maximum compressive load at which the column is start to crushing.

Buckling load ⇒ The maximum axial compressive load at which the column is start to buckle.

Euler's Formula:-

The buckling load for long column is calculated by using Euler's formula.

$$\boxed{P_e = \frac{\pi^2 EI}{l_e^2}}$$

where, E = Young's modulus of elasticity

I = Least moment of inertia

l_e = Effective Length

Elasticity ⇒ The material which has ability to return it's original shape after removing the external load.

Isotropic ⇒ If the property of the material is same along all direction is known as isotropic.

Homogeneous ⇒ which object is made up of a single material.

or

If the property of the material is same in every crossection.

Effective length ⇒ when the both end of the column is pinjointed, it's length is called effective length.

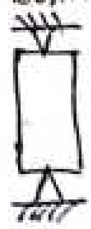
Assumptions made in Euler's formula:-

1) Material or column is should be elastic, isotropic and homogeneous.
2) The column is perfectly straight and of uniform lateral dimensions.
3) Load is applied axially and passes through the centroid.
4) This is valid for long column only.

Effective length :-

~~Effective length of a column of given dimensions~~

① Both ends are hinged (Pin jointed)

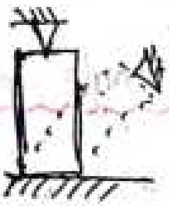


$\boxed{l_e = l}$

② One end of the column is ~~hinged~~ fixed and other end is free -

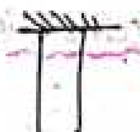
$\boxed{l_e = 2l}$

③ One end is fixed and other end is hinged -



$\boxed{l_e = \frac{l}{\sqrt{2}}}$

④ Both ends are fixed -



$\boxed{l_e = \frac{l}{2}}$

2012 Application of Eulers formula :- Imp

①. Both ends are hinged :-

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

but here $l_e = l$

So, $P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$

$$\boxed{P_E = \frac{\pi^2 EI}{l^2}}$$

(2) If one end is fixed and other end is free -

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

But here $l_e = 2l$

so, $P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}$

$$\Rightarrow \boxed{P_E = \frac{\pi^2 EI}{4l^2}}$$

(3) If one end is fixed and other end is hinged -

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

But here $l_e = \frac{l}{\sqrt{2}}$

so, $P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI}{l^2}$

$$\Rightarrow \boxed{P_E = \frac{2\pi^2 EI}{l^2}}$$

(4) Both ends are fixed -

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

But here $l_e = \frac{l}{2}$

$P_E = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}$

$$\Rightarrow \boxed{P_E = \frac{4\pi^2 EI}{l^2}}$$

Slenderness ratio (λ):-

The ratio of effective length to minimum radius of gyration is called as slenderness ratio.

i.e. $\boxed{\text{Slenderness ratio} = \frac{\text{Effective length of column } (l_e)}{\text{Minimum radius of gyration } (k)}}$

$$\Rightarrow \boxed{\lambda = \frac{le}{K}}$$

$$\lambda = \frac{le}{K}$$

$$I = AK^2$$

$$\Rightarrow K = \sqrt{\frac{I}{A}}$$

$$P_E = \frac{\pi^2 EI}{le^2} = \frac{\pi^2 EAK^2}{le^2} = \frac{\pi^2 EA}{\left(\frac{le}{K}\right)^2} = \frac{\pi^2 EA}{\lambda^2} \left[\because \lambda = \frac{le}{K}\right]$$

$$\Rightarrow \boxed{P_E = \frac{\pi^2 EA}{\lambda^2}}$$

$\therefore \lambda < 32 \rightarrow$ Short column.

$\therefore 32 < \lambda \not< 120 \rightarrow$ Intermediate column

$\therefore \lambda > 120 \rightarrow$ Long column.

Classification of columns:-

① Short columns:- Columns having their length ≤ 8 times their respective diameter or having slenderness ratio <32 are known as short columns.

When the short columns are subjected to compressive loads, there buckling is negligible or very small as compare to the crushing stress, therefore the short columns are assumed to fail under crushing only. fails due to crushing only

② Intermediate columns:- The columns having their length varying from 8 times to 30 times their respective diameters or having slenderness ratio greater than 32 but less than 120 are known as intermediate columns. These columns are subjected to both crushing and buckling stress. So, the columns may fail either due to crushing or buckling.

fails due to ~~both~~ either buckling or crushing

(3) Long column:- The columns having their length greater than 30 times their respective diameter or having slenderness ratio greater than 120 are known as Long columns.

When this columns are subjected to compressive stress, buckling stress is pre-dominant and crushing stress is negligible, so the columns will fail due to buckling only.

Fails due to buckling only.

Radius of Gyration:- (K)

It may be defined as the square root of the ratio of moment of inertia of body about it's CG to it's crossectional area.

i.e. $I = AK^2$

$\Rightarrow K = \sqrt{\frac{I}{A}}$

$$K_{rectangle} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{\frac{db^3}{12}}{d \times b}} = \sqrt{\frac{b^2}{12}} = \frac{b}{\sqrt{12}} = \frac{b}{2\sqrt{3}}$$

$$\Rightarrow \boxed{K_{rectangle} = \frac{b}{2\sqrt{3}}}$$

$$K_{circle} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}}} = \sqrt{\frac{\pi d^4}{64_{16}} \times \frac{4}{\pi d^2}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}$$

$$\Rightarrow \boxed{K_{circle} = \frac{d}{4}}$$

$$\cancel{K_{square} = \sqrt{\frac{I}{A}}}$$

$$K_{square} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{\frac{a^4}{12}}{a^2}} = \sqrt{\frac{a^2}{12}} = \frac{a}{2\sqrt{3}}$$

$$\Rightarrow \boxed{K_{square} = \frac{a}{2\sqrt{3}}}$$

Numericals:-

Q) A solid steel rod of 60 mm diameter and 25m Length is used in a column. Determine the safe load that the column can carry without buckling for the following condition.

$E = 2 \times 10^5 \, N/mm^2$ and factor of safety = 3

① Both the ends are hinged

② Both the ends are fixed

Soln

Given data

$d = 60\,mm = 0.06\,m$

Length $l = 25\,m$

$E = 2 \times 10^5 \, N/mm^2$ $\qquad 1mm = 10^{-3} m$

$= 2 \times 10^5 \dfrac{N}{10^{-6} m^2}$ $\qquad 1mm^2 = (10^{-3})^2 m^2 = 10^{-6} m^2$

$= 2 \times 10^{5-(-6)} \dfrac{N}{m^2}$

$\Rightarrow E = 2 \times 10^{11} \, N/m^2$

We know that → Columns having greater than 30 times their respective diameter

$d = 30 \times 60 = 1800\,mm = 1.8\,m$

Length $= 25m > 30d$ [∵ Long column]

① In case of both ends are hinged

$P_{cr}/_{bcr} = \dfrac{\pi^2 EI}{le^2} = \dfrac{\pi^2 EI}{le^2}$ $\left[\because MI \text{ of circle} = \dfrac{\pi d^4}{64} \right.$ $\dfrac{\pi \times (0.06)^4}{64} = 6.36 \times 10^{-7} m$

$= \dfrac{\pi^2 \times (2 \times 10^{11}) \times (6.36 \times 10^{-7})}{(25)^2}$

$= 2008\,N$

$$\text{Safe load} = \frac{P_{cr}/b_{ck}}{\text{F.O safety}} = \frac{2008}{3} = 669.3\,N$$

(2) Both ends are fixed -

$$P_{cr}/b_{ck} = \frac{\pi^2 EI}{le^2} = \frac{\pi^2 EI}{(\frac{l}{2})^2} = \frac{4\pi^2 EI}{l^2} = 4 \times 2008 = 8032\,N$$

$$\text{Safe load} = \frac{P_{cr}/b_{ck}}{\text{F.O safety}} = \frac{8032}{3} = 2677\,N.$$

Shear force and Bending moment

Stresses in beams due to bending:-

Beam → It is a horizontal structure member which is design to carry the external load and moments acting on it safely to the adjucent columns.

or

Beam may also be ~~desig~~ defined as a horizontal structural member that bends. It means, a beam can only fail due to bending only.

→ When a beam is subjected to bending it is subjected to shear force and bending moment.

→ Due to shear force, shear stress is developed and due to bending moment, bending moment developed.

Bending equation or flexure equation :-

$$\boxed{\frac{M}{I} = \frac{F}{Y} = \frac{E}{R}}$$ → It's known as bending equation or flexure equation.

[Simple trick → Elezabeth Rani May I Follow You]

Some Assumptions :-

(i) Plane section remain plane even after bending takes place i.e. no twisting or wrapping occurs.

or

In other words, strain profile and diagram is linear.

(ii) Material of the beam is elastic, homogeneous, isotropic and obeys Hooke's law.

(iii) Beam is subjected to bending only.

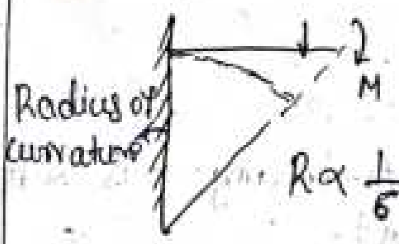
(iv) Every layer of the beam expand or contract freely.

(v) The value of Young's modulous of elasticity is same through out the section.

i.e. $\boxed{E_{tension} = E_{compression}}$

(vi) The beam bends in the shape of a arc of a circle

(vii) The radius of curvature of beam is very large as compare to it's transverse direction [i.e. deflection is too small]



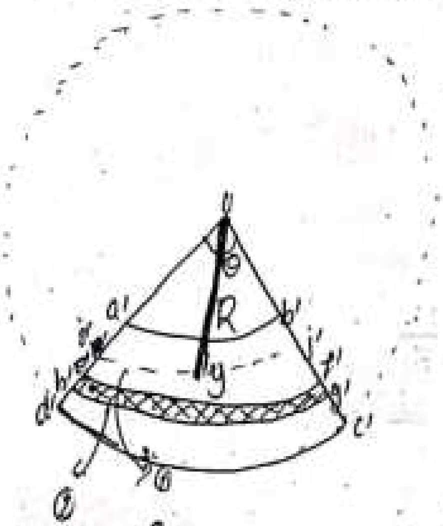
$R \propto \frac{1}{\delta}$

Bending of beam:-

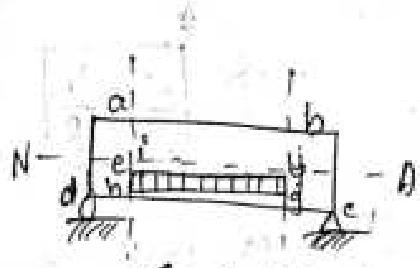
1- Normal → Bending stress → Bending moment

2- Shear stress → Shear force

Derivation of bending equation:-



(After bending)



(Before bending)

- Let us consider a small portion of beam called (abcd) when subjected to external loading supposed it's getting deflected into the shape (a'b'c'd'), let us assume a small strip of beam (efgh) at a distance of (y) below the natural axis (ij) and after bending it get (e'f'g'h').

So the strain developed in the layer ef

$$\boxed{\varepsilon_{ef} = \frac{\text{Change in Length}}{\text{Actual Length}} = \frac{e'f' - ef}{ef}}$$

here $e'f' = (R+y)\theta$

$ef = gh = ij = i'j'$ [Since there is no change in length along natural axis ij]

$ef = i'j' = R\theta$

So, $\varepsilon_{ef} = \frac{e'f' - ef}{ef}$

$$= \frac{(R+y)\theta - R\theta}{R\theta}$$

$$= \frac{R\theta + y\theta - R\theta}{R\theta}$$

$$= \frac{y\theta}{R\theta}$$

$$\Rightarrow \varepsilon_{ef} = \frac{y}{R}$$

So $\boxed{\varepsilon = \frac{y}{R}}$

But we know $\varepsilon = \frac{F}{E}$

$$\therefore \frac{F}{E} = \frac{y}{R}$$

$$\Rightarrow \boxed{\frac{F}{y} = \frac{E}{R}} \text{ —— eqn (1)}$$

Thrust acting on gh

$= \sigma \cdot dA$ [here area = dA]

$= \frac{E}{R} \cdot y \times dA$ [e.g. area = dA]

Moment of thrust acting on (dA) due to small strip gh

$$M = \left(\frac{E}{R} \cdot y \cdot dA\right) \times y$$

$$= \frac{E}{R} y^2 \, dA$$

So total amount on the beam is known as second moment of area and also known as M.I.

$$= \int_0^A \frac{E}{R} \cdot y^2 \, dA$$

$$\Rightarrow M = \frac{E}{R} \int_0^A y^2 dA = I$$

$\cancel{\Rightarrow \frac{E}{R} \ldots \frac{M}{I} = \frac{E}{R}}$

$$\Rightarrow \quad = \frac{E}{R} \cdot I$$

$$\Rightarrow \boxed{\frac{M}{I} = \frac{E}{R}} \text{ ———— } eq^n \text{ (2)}$$

comparing both equation we get

$$\boxed{\frac{F}{y} = \frac{E}{R} = \frac{M}{I}}$$ → Bending Equation or (proved) Flexure equation

$$\boxed{\frac{E}{y} = \frac{M}{I} = \frac{F/\sigma}{y}}$$

where,

M → Moment of resistance or bending moment

I → Moment of inertia

F/σ → Bending stress

y → Distance from Neutral axis

E → Young's modulous of elasticity

R → Radius of curvature

Bending stress distribution :-

We know that $\frac{M}{I} = \frac{F}{y} = \frac{E}{R}$

So $\frac{M}{I} = \frac{F}{y}$

$$\Rightarrow \boxed{F = \frac{M}{I} \cdot y}$$

At a particular section - M = constant & I = constant we can say that bending stress (F) is only dependent on 'y' (distance from natural axis)

i.e.

$$F \propto y$$

Since, the value of 'y' is maximum at the top-most and bottom-most fibre. Bending stress will be maximum at top-most and bottom-most fibre.

Questions

1) In compression zone bending stress is maximum at which point?-

(a) Top-fibre ✓ (c) Neutral point

(b) Bottom fibre (d) Non ~~op~~ up the above

2) In tensile zone bending stress is maximum at which point?-

(a) Top-fibre (c) Neutral point

(b) Bottom fibre ✓ (d) Non up above

3) In a beam bending stress is maximum at which point?-

(a) Top-fibre (c) Neutral point

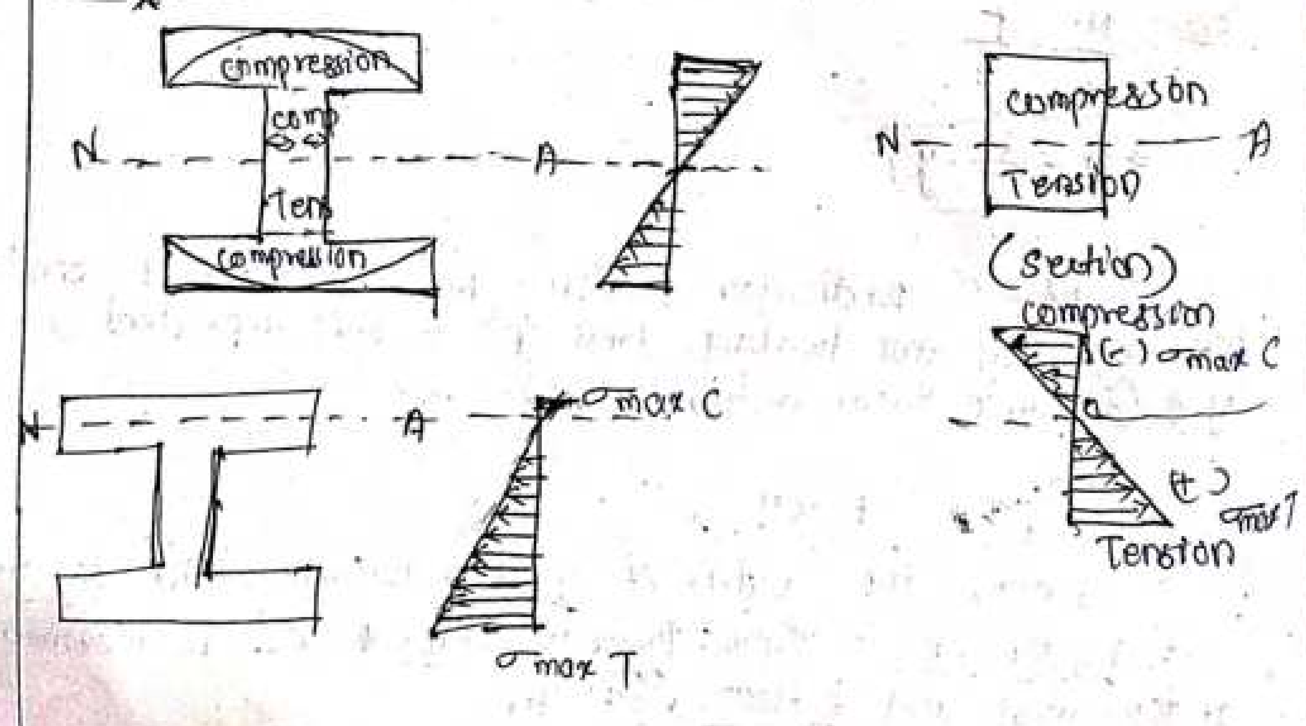
(b) Bottom fibre (d) Both a & b ✓

4) In a beam bending stress is zero at which point?

(a) Top-fibre (c) Neutral point ✓

(b) Bottom fibre (d) Non oup the above

—x—



$$\sigma_{max} = \frac{M}{I} y_{max}$$

$$\Rightarrow \sigma_{max} = \frac{M}{\left(\frac{I}{y_{max}}\right)} \rightarrow z \text{ (section modulus)}$$

$$\Rightarrow (\downarrow) \sigma_{max} = \frac{M}{z(\uparrow)}$$

$$\boxed{z = \frac{I}{y_{max}}} \rightarrow \text{section modulus}$$

Section modulus(z):-

we know that

$$\sigma_{max} = \frac{M}{I} y_{max}$$

$$\Rightarrow \sigma_{max} = \frac{M}{\frac{I}{y_{max}}}$$

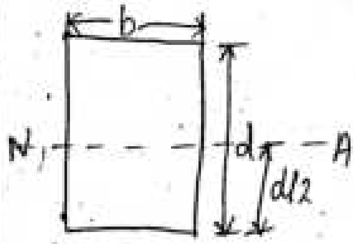
$\frac{I}{y_{max}}$ is known as section modulus.

and it is ~~known as sectio~~ denoted by 'z', so we can say $\sigma_{max} = \frac{M}{z}$

where z = section modulus from where we can say that $\left(\sigma \propto \frac{1}{z}\right)$

Section modulus of different sections:-

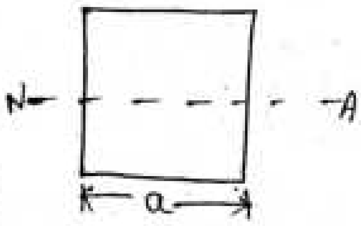
a- Rectangle:-



$$z = \frac{I}{y_{max}} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^{\cancel{3}\,2}}{\cancel{12}\,6} \times \frac{\cancel{2}}{\cancel{d}} = \frac{bd^2}{6}$$

$$\Rightarrow \boxed{z = \frac{bd^2}{6}}$$

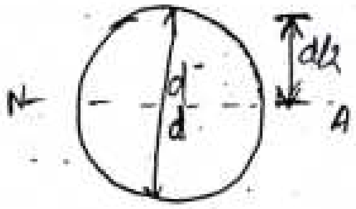
b - Square :-



$$z = \frac{I}{y_{max}} = \frac{\frac{a^4}{12}}{a/2} = \frac{a^{\cancel{4}3}}{12} \times \frac{2}{\cancel{a}} = \frac{a^3}{6}$$

$$\Rightarrow \boxed{z = \frac{a^3}{6}}$$

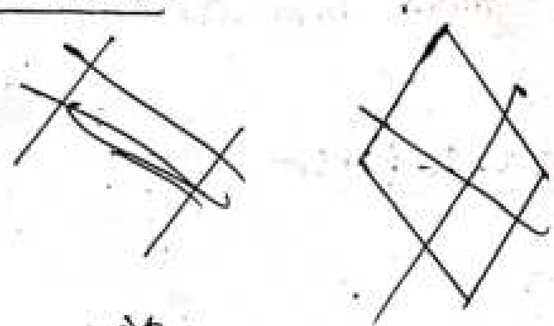
c - Circle :-

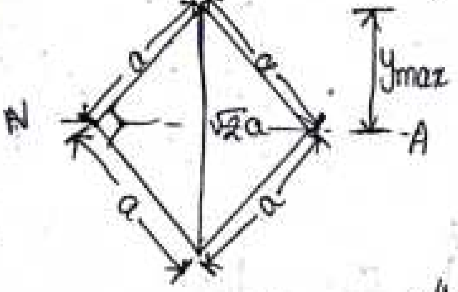


$$z = \frac{I}{y_{max}} = \frac{\pi d^{\cancel{4}3}/\cancel{64}\,32}{\cancel{d}/\cancel{2}} = \frac{\pi d^3}{32}$$

$$\Rightarrow \boxed{z = \frac{\pi d^3}{32}}$$

d - Diamond :-





$$Z_{dia} = \frac{I}{y_{max}} = \frac{a^4/12}{\left(\frac{\sqrt{2}a}{2}\right)} = \frac{a^4/12}{\frac{\sqrt{2}(\sqrt{2}a)}{\sqrt{2}\cdot 2}}$$

$$= \frac{a^4}{12} \times \frac{\sqrt{2}}{a} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{a^{4/3}}{\cancel{12}_6} \times \frac{\cancel{2}}{a\sqrt{2}}$$

$$= \frac{a^3}{6\sqrt{2}}$$

$$\Rightarrow \boxed{Z = \frac{a^3}{6\sqrt{2}}}$$

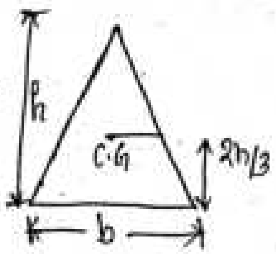
$$\frac{Z_{square}}{Z_{diamond}} = \frac{a^3/6}{a^3/6\sqrt{2}} = \frac{\cancel{a^3}}{\cancel{6}} \times \frac{\cancel{6}\sqrt{2}}{\cancel{a^3}} = \sqrt{2}$$

$$\Rightarrow \boxed{\frac{Z_{square}}{Z_{diamon}} = \sqrt{2}}$$

$$\Rightarrow \boxed{Z_{square} = \sqrt{2} \times Z_{diamond}}$$

$$\Rightarrow \boxed{Z_{square} = 1.414 \times diamond}$$

e- Triangle :-



$$Z = \frac{I}{y_{max}} = \frac{\frac{bh^3}{36}}{2h/3} = \frac{bh^{\cancel{3}\,2}}{\cancel{36}_{12}} \times \frac{\cancel{3}}{2h} = \frac{bh^2}{24}$$

$$\Rightarrow \boxed{Z = \frac{bh^2}{24}}$$

⇒ The value of sectional modulus is maximum in case of square and rectangle.

⇒ 'I' section is the strongest section.

Hence $Z_{I\ section} > Z_{rectangle}$ or $Z_{square} > Z_{circle}$

⇒ Beam of a bridge is called gardor.

Strength of the beam:-

The strength of beam section is devided by the section modulus (Z) of the section.

$$\sigma = \frac{M}{Z}$$

Higher the value of section modulus (Z) of the sectio lower is the bending stress (σ). So the setion is more stron

Q- A steel wire of 5mm diameter is bent into a circular shape of 5m radius. Determine the maximum stress ~~induced~~ induced in wire.

Ans

Data given

$E = 200\ GPa = 200 \times 10^9\ N/m^2$

diameter $= 5mm$

radius $= 5m$

$\sigma_{max} = ?$

We know,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$= \frac{\sigma}{y} = \frac{E}{R}$$

$$\therefore \sigma = \frac{E}{R} \cdot y_{max}$$

$$= \frac{200 \times 10^9\ N/m^2}{5m} \times (2.5 \times 10^{-3})m$$

$$= 10^8\ N/m^2$$

Q) An alloy wire of 5mm diameter is required to wound around a ~~dreem~~ drum of 3m diameter the maximum stress in the wire is not to exceed 200Mpa. And the value of E of the alloy.

Ans

Data given

diameter of alloy wire = 5 mm

Radius of wire = 3/2 = 1.5 m

σ_{max} = 200 mpa

E = ?

We know that

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{E}{R} = \frac{\sigma}{y}$$

$$\Rightarrow E = \frac{\sigma}{y} \times R$$

$$= \frac{200 \times 10^6 \frac{N}{m^2}}{2.5 \times 10^{-3}} \times 1.5$$

$$= 1.2 \times 10^{11} \frac{N}{m^2}$$

$$= 120 \text{ GPa}$$

Shear stress in beams due to shear force :-

Shear stress is denoted as (τ) tau.

$$\boxed{\tau = \frac{FA \times \bar{y}}{Ib}}$$

→ Bending stress is developed due to bending moment.

→ Shear stress is developed due to shear force.

→ Maximum Bending stress is developed at outermost fibre. but incase of shear stress maximum at neutral axis.

→ Normal stress is denoted by 'σ' but shear stress is denoted by 'τ'.

Shear stress is developed in the beam to shear force may be given by

$$\boxed{\tau = \frac{FA\bar{y}}{Ib}}$$

where,

$F \rightarrow$ Shear force

$A \rightarrow$ Area either below / above the point under considiration

$\bar{y} \rightarrow$ Distance of C.G of section under considiration from neutral axis.

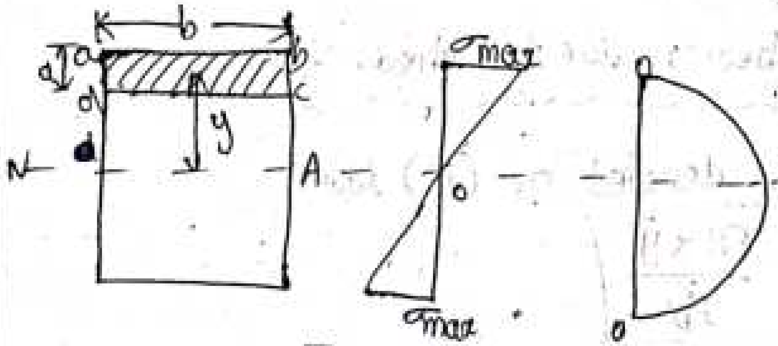
$b \rightarrow$ Breadth of section

$I \rightarrow$ Moment of inertia above neutral axis.

Note

Shear-stress varies in the section along its depth only and remains constant along its width.

Rectangular section



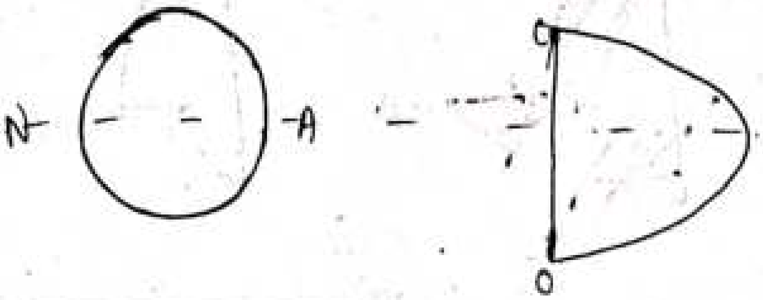
$$\boxed{\tau_{max} = \frac{3}{2} \times \tau_{avg}}$$

[Bending stress developed] [Shear stress developed]

→ Shear stress is maximum at neutral axis.

$$\boxed{\tau_{max} = \frac{3}{2} \times \tau_{avg}}$$

b) circular section :-

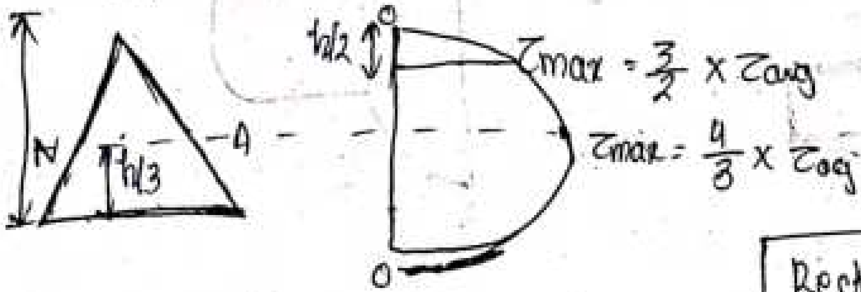


$$\boxed{\tau_{max} = \frac{4}{3}\,\tau_{avg}}$$

[redacted]

$$\boxed{\tau_{max} = \frac{4}{3} \times \tau_{avg}}$$

c) Triangular section :-

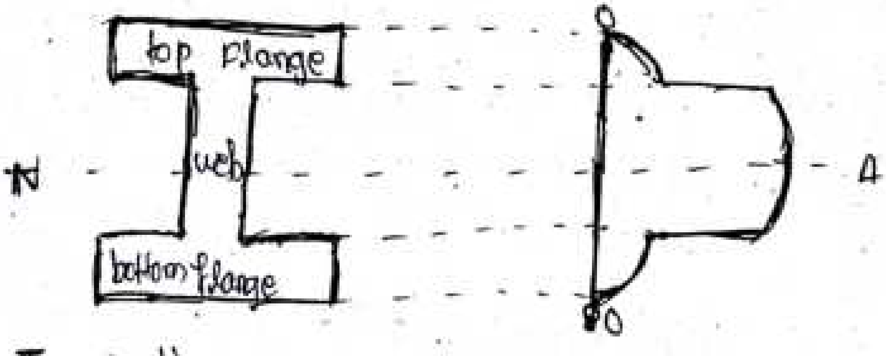


$$\tau = \frac{FA\bar{y}}{Ib}$$

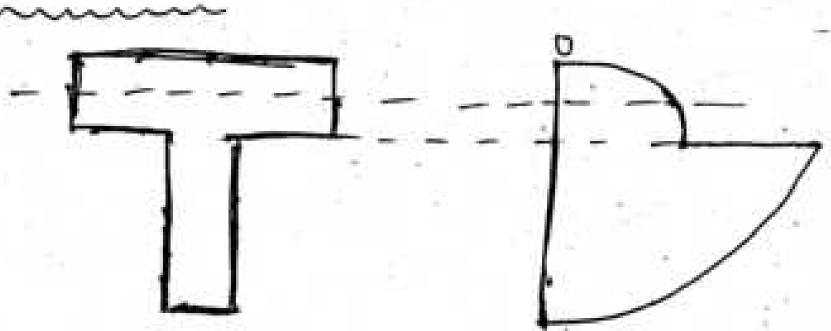
$$\boxed{\tau \propto \frac{1}{b}}$$

Rectangle = $\tau_{max} = \frac{3}{2} \times \tau_{avg}$
Circle = $\tau_{max} = \frac{4}{3} \times \tau_{avg}$
Triangle = $\tau_{max} = \frac{4}{3} \times \tau_{avg}$ at NA

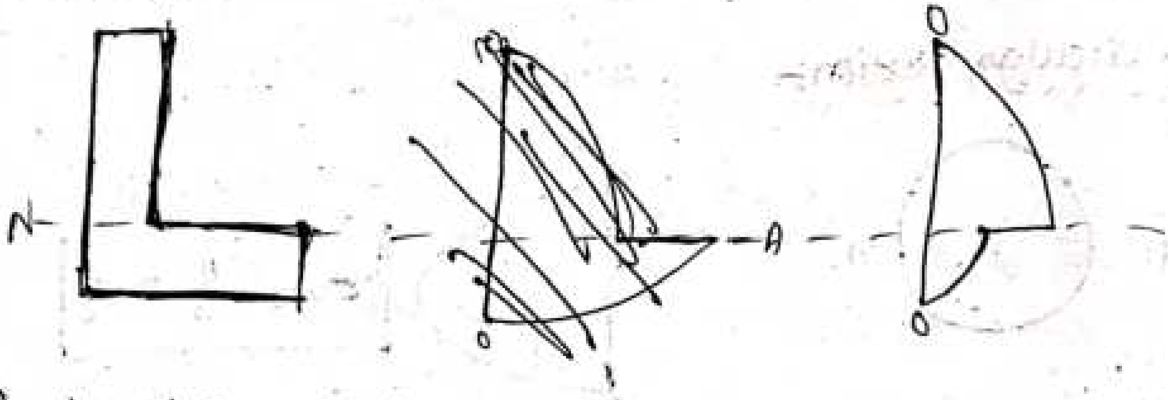
d) I-section :-



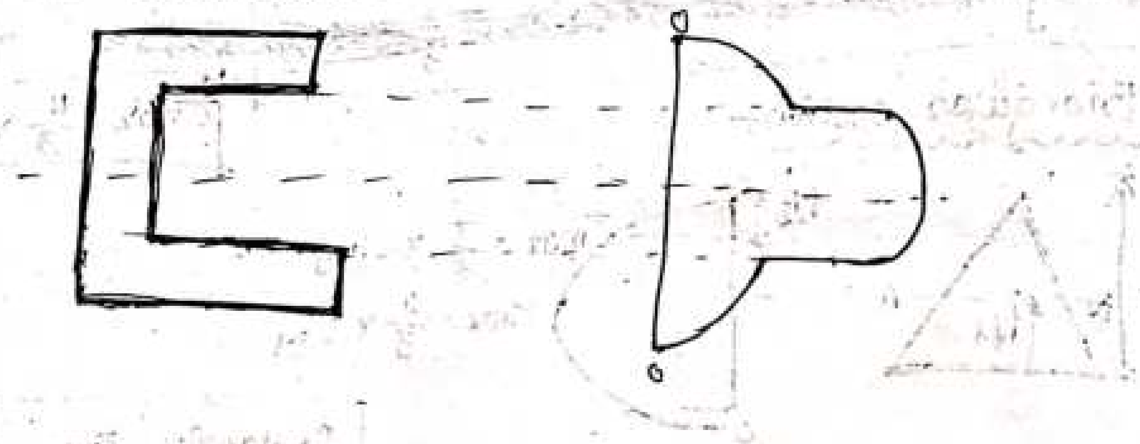
e) T-section :-



f) L - section:-



g) channel / C- section:-



Torsion of shafts

$\boxed{\text{Torque} = F \times \text{distance}}$

$\boxed{\text{Moment} = F \times \perp\text{ar distance}}$

Difference between Bending moment and Torque:-

Bending Moment	Torque
→ Moment is applied along the tranverse axis of the member.	→ Moment is applied along the longitudinal axis.
→ Bending of section occurs.	→ Rotation of section occurs.
→ Normal stress is developed.	→ Shear stress is developed.

Shaft → The circular member which undergo torsion.

Torsion:-

Due to application of torque rotation of the shafts occurs, this rotating effect of torque on shafts is known as torsion.

$$\boxed{\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}}$$ → Equation of torque.

Assumption of pure torsion:-

Pure torsion:- It means only torque is applied on the shaft no bending moment is acting on shaft is known as pure torsion.

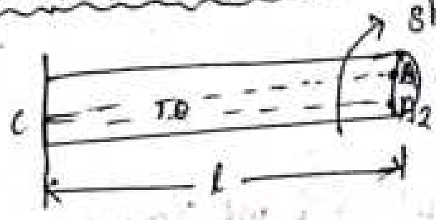
Assumption:-

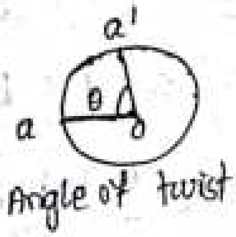
i) The material is elastic, homogeneous and isotropic.

ii) Crossection of shafts, which is circular uniform and plane before twisting remains planes even after, twisting occurs.

iii) All diameters of the normal crossection before twist remains plane and straight, even after the twist occurs.

Derivation of torsion equation :-





Let us consider a circular shaft of length (L) is which fixed at one ends and subjected to torque (T) at the free end.

Let us consider a point (a) on the shaft is getting shifted to (a') after application of (T), so, the angle created due to twisting along the length of the section is $\angle ACA'$

Let $\angle ACA' = \phi$ shear strain

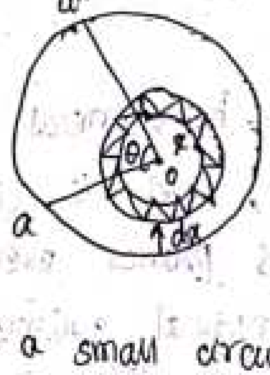
Let θ be the angle created at the centre of the shaft due to twisting of the shaft i.e. ($\angle AOA' = \theta$ = Angle of twist

Then length of arc $= AA' = L \times \phi = R\theta$

So, $L\phi = R\theta$

$\Rightarrow (L \times \frac{\tau}{C} = R\theta)$ ($\because$ = shear strain (ϕ)) = $\frac{\text{Shear stress}}{\text{Moment of rigidi}}$ $= \frac{\tau}{C}$

$\boxed{\frac{\tau}{R} = \frac{C\theta}{L}}$ → Equation - (1)



Let us consider a small circular strip of the shaft at a distance (x) from the centre of shaft. Let the thickness of small strip of the shaft be (dx).

$\Rightarrow$ Area of the small strip (da) $= 2\pi x \times dx$

shear force $=$ shear stress $\times$ Area

$$= \tau x \times 2\pi x \times dx$$

So, Twisting moment of torque (T) $= \tau x \times 2\pi x \times dx \times x$

here, $\frac{\tau x}{\tau} = \frac{x}{R}$

$$\Rightarrow \tau x = \boxed{\frac{\tau}{R} \times x}$$

So, Torque on the small strip $= \tau x \times 2\pi x dx \times x$

$$= \frac{\tau}{R} \cdot x \times 2\pi x \, dx \times x$$

$$= \frac{\tau}{R} \times 2\pi \times x^3 dx$$

$$= \frac{2\pi}{R} \times x^3 dx \times \tau$$

Hence

Torque acting on the entire shaft

$$= \int_0^R \frac{2\pi}{R} \cdot x^3 dx \cdot \tau$$

$$= \frac{2\pi}{R} \cdot \tau x \int_0^R x^3 dx$$

$$= \frac{2\pi}{R} \tau x \left[\frac{x^4}{4}\right]_0^R$$

$$= \frac{2\pi}{R} \times \tau \times \left[\frac{R^4}{4} - 0\right]$$

$$= \frac{2\pi}{R} \cdot \tau \cdot \frac{R^4}{4}$$

$$= 2\pi \cdot \tau \cdot \frac{R^3}{4}$$

So,
$$T = 2\pi \cdot \tau \cdot \frac{R^3}{4}$$
$$= 2\pi\tau \times \frac{(D/2)^3}{4}$$
$$= 2\pi\tau \times \frac{D^3}{32}$$

From $\perp$ar axis theorem known,

$$I_{xx} = I_{yy} + I_{zz}$$
$$I_p = \frac{\pi d^4}{64} + \frac{\pi d^4}{64}$$
$$\Rightarrow I_p = \frac{\pi d^4}{32}$$

Again we known

$$T = 2\pi\tau \frac{D^3}{32} \times \frac{D}{D}$$
$$= 2\pi \times \tau \times \frac{D^4}{32} \times \frac{1}{D}$$
$$= 2\tau \times \frac{\pi D^4}{32} \times \frac{1}{D}$$
$$= 2 \times \tau \times I_p \times \frac{1}{D}$$
$$= \tau \times I_p \times \frac{1}{(D/2)}$$
$$= \tau \times I_p \times \frac{1}{R}$$

So

$$\cancel{T = \frac{T}{I_p} = \frac{\tau}{R}}$$

$$T = \tau \times \frac{I_p}{R}$$

$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R}} \quad \text{eqn} — (2)$$

compairing both equation ① & ② we get

$$\boxed{\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}}$$

(proved)

This equation known as torson equation where,

T = Torque

I_p = Polar moment of intertia

τ = Shear stree

R = Radius of shaft

(G & C) = Modulus of Rigidity

θ = Angle of twist

l = Length of shaft

ϕ = Shear strain

Q) A circular shaft of 50mm diameter is to required to transmite torque from one to another. Find the shape torque which shaft can transmite if the is not exceed 40MPa.

Soln.

Given that,

$$d = 50mm, \ \tau = 40\,MPa = 40 \times 10^6\ N/mm^2$$

we know

equation $\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$

$$= \frac{T}{I_p} = \frac{\tau}{R}$$

$$\Rightarrow \frac{T}{\frac{\pi d^4}{32}} = \frac{40}{25}$$

$$T = \frac{\pi \times (50)^4}{32} \times \frac{40}{25}$$

$$= 981747.70\ Nmm.$$

Q) A solid shaft is to transmit a torque of 10KW is shearing stress is not to exceed 45MPa. Find the minimum diameter of the shaft.

Soln

Given that

$T = 10\text{ KN}\cdot m = 10 \times 10^3 N \times 10^3 mm = 10^7 Nmm,\ \tau = 45MPa.$

we know that

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R}}$$

$T = 10^7\ N.mm$

$\tau = 45$

$R = d/2$

$$\frac{T}{I_p} = \frac{\tau}{R}$$

$$\Rightarrow \frac{10^7}{\frac{\pi d^4}{32}} = \frac{45}{d/2}$$

$$\Rightarrow \frac{32 \times 10^7}{\pi d^4} = \frac{90}{d}$$

$$\Rightarrow d = \frac{\sqrt[3]{32 \times 10^7}}{90 \times \pi}$$

$$\Rightarrow d = 10^4 mm$$

Tortional strength of hollow circular shaft:-

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow T = \frac{\tau}{R} \cdot I_p$$

$$T = \frac{\tau}{(d/2)} \times \frac{\pi}{32}(D^4 - d^4)$$

$$T = \frac{2\tau}{D} \times \frac{\pi}{32}(D^4 - d^4)$$

$$\boxed{T = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau}$$

[d = Internal diameter

D = External diameter]

Q) A hollow circular shaft of external & internal diameter 80mm and 50mm is required to transmite torque one end to other and what is shaped of it the allowable shear stress is 45 MPa.

Soln

Given that

$D = 80mm$, $d = 50mm$, $\tau = 45 MPa = 45 N/mm^2$

we know,

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R}}$$

$$T = \frac{\tau}{D/2} \times \pi/32 \, (D^4 - d^4)$$

$$= \frac{45}{80/2} \times \frac{\frac{22}{7}}{32} (80^4 - 50^4)$$

$$\Rightarrow T = 3833602.067$$

Polar Modulus :-

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow \frac{T}{I_p} = \frac{\tau}{R}$$

∴ It is denoted as $\frac{I_p}{R} = \frac{J}{R}$ and it is also known as torsional modulus of shaft.

Torsional stiffness :-

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow \frac{T}{\theta} = \frac{CI_p}{l}$$

It is known as torsional stiffness, it may be defined as the amount of torque required to produce one radian angle of twist.

Stiffness:-

Force required to reduce unit deformation is known as stiffness.

∴ The product of modulus of elasticity 'E' and moment of inertia "I" is known as flexural rigidity.

$$\frac{M}{I} = \frac{F}{y} = \frac{E}{R}$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow \boxed{M = \frac{EI}{R}}$$

[EI is known as flexural rigidity]

∴ The product of shear modulus of rigidity "C" and polar moment of inertia "Ip" is known as torsional rigidity.

$$\boxed{CI_p = \text{Torsional Rigidity}}$$

Torsional flexibility:-

The inverse of torsional stiffness is known as ~~tors~~ torsional flexibility.

$$T.F = \frac{1}{\text{Torsional stiffness}} = \frac{1}{CI_p/l} = \frac{l}{CI_p}$$

Q) Find the angle of twist per meter length of a hollow shaft of 100mm external diameter and 60 mm internal diameter. If the shear stress is not to exceed 85 Mpa take (C = 85 GPa).

Soln

We known that

$$T = \frac{\pi}{16} \frac{(D^4 - d^4)}{D} \tau$$

$l = 1m$

$\tau = 85$ MPa , C = 85 GPa

$= 85 \text{ N/mm}^2$ $= 85 \times 10^3 \text{ N/mm}^2$

so,
$$\frac{\tau}{R} = \frac{C\theta}{l}$$

$$\Rightarrow \frac{35}{(D/2)} = \frac{85 \times 10^3 \times \theta}{10^3}$$

$$\Rightarrow \frac{35}{50} = \frac{85 \times 10^3 \times \theta}{10^3}$$

$$\Rightarrow \theta = \frac{35}{50 \times 85}$$

$$= 8.23 \times 10^{-3} \text{ rad}$$

$$= 8.23 \times 10^{-3} \times \frac{180}{\pi}$$

$$\Rightarrow \theta = 0.471^\circ$$

Stress :-

Internal resisting force per unit area is called stress.

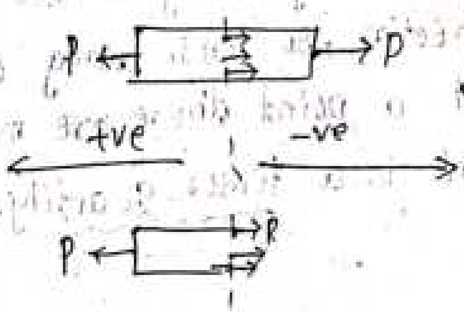
⇒ Internal resisting force per crossectional unit area is called stress.

$$\sigma_{max} = \frac{P}{A_{min}}$$

S.I Unit $= \frac{N}{m^2} = \text{Pascal}, \frac{N}{mm^2}, \frac{dyne}{cm^2}, \frac{kgf}{cm^2}$

$\frac{N}{mm^2}$

In structural Mechanics $\boxed{F_{net} = 0}$



$\Sigma F_{net} = 0$

$\Rightarrow -P + R = 0$

$\Rightarrow \boxed{R = P}$

$$\boxed{\text{stress}(\sigma) = \frac{R}{A} = \frac{P}{A}}$$

Complex Stress

When the body is subjected to forces acting along one direction only the analysis of stress is quite simple and easy, but when forces are acting on the body along many directions the analysis is ~~~~ complex hence complex stress is analysis is used to determine the failure pattern of the body.

Representation of stress:-

There are two kinds of stress -

① Normal stress

② Shear stress

① Normal stress (σ):-

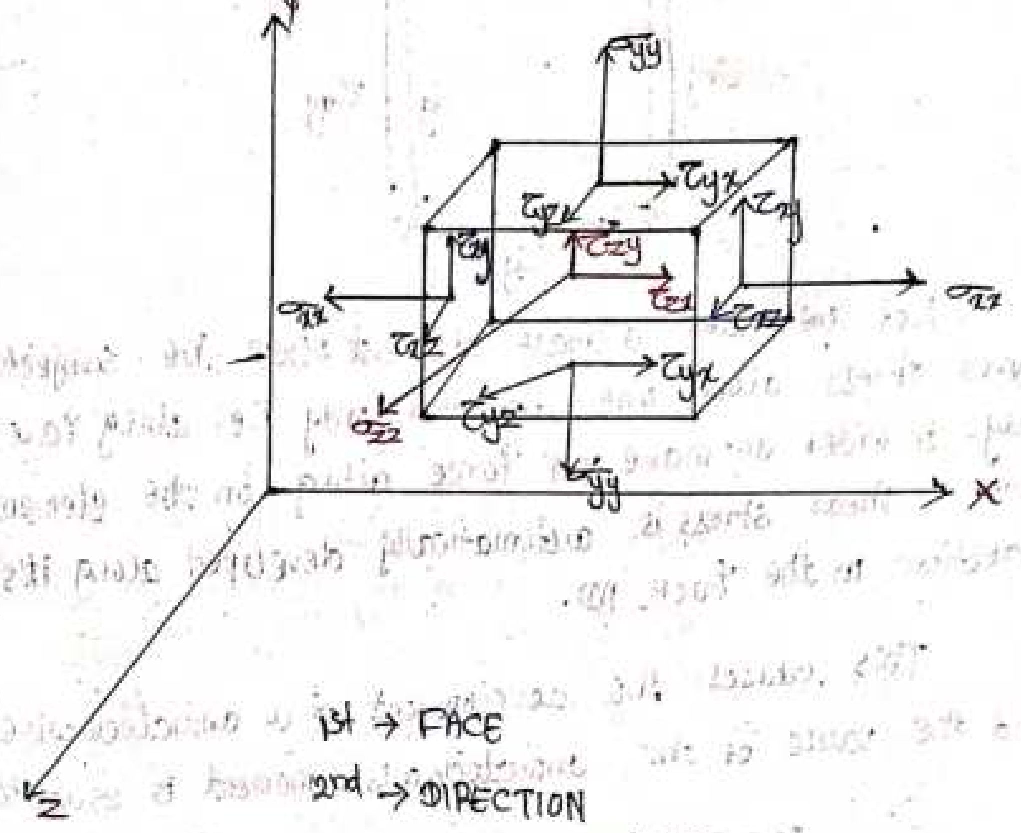
When the force is applied perpendicular to the surface of the body, the stress induced in the body is known as normal stress and it is denoted with 'σ'.

② Shear stress (τ):-

When the force is applied parallel to the surface of the body, the stress induced in the body is known as shear stress and it is denoted with 'τ' symbole.

When stress is acting along a face it has both magnitude and direction, so stress along a face is a vactor quantity. But w.r.t a point there are many faces possible so stress at a point is a tensor quantity.

~~So stress quantity is represented with the help of too subscripts, where the first subscript represent the fa~~



So stress quantity is represented with the help of too subscripts, where the first subscript represent the face and the second subscript represents the direction.

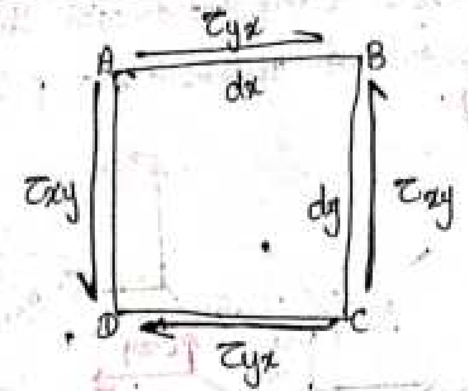
2 subscripts [σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{xz}, σ_{yz}

[τ_{xx}, τ_{yy}, τ_{zz}, τ_{xy}, τ_{yz}, τ_{xy}

5 or 2 Mark

Complementary shear stress:-

If shear stress is acting along one direction only, shear stress is automatically developed in all other Lar directions. This nature of shear stress is known as complementary shear stress.



-Let us take a small element ABCD be subjected to shear stress along one direction only i.e. along face BC only. In order to make net force acting on the element to be zero, shear stress is automatically developed along it's opposite direction in the face AD.

This causes the development of a anticlockwise moment and the value of this anticlockwise moment is equal to

$$M = \tau \times (dy \times 1)\, dx$$

In order to make net moment acting on the element is equal to zero, automatically a clockwise moment will be developed along it's opposite direction along the face AB & CD i.e.

$$M_{net} = 0$$

$$\Rightarrow \tau \times (dy \times 1) \times dx - (\tau \times (dy \times 1)\, dx) = 0$$

→ Magnitude of anticlockwise couple $= \tau_{xy} \times (dy \times 1) \times dx$

→ Magnitude of clockwise couple $= \tau_{yx} \times (dx \times 1) \times dy$

so, $\tau_{xy} \times (dy \times 1) \times dx = \tau_{yx} \times (dx \times 1) \times dy$

$$\Rightarrow \boxed{\tau_{xy} = \tau_{yx}}$$

Elementary stress can be represented with the help of matrix also and it is given by

Stress Tensor $= \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$

In the above matrix the total no of stress component is equal to '9', no of normal stress component '3', no of shear stress component is equal to '6' and no of independent stress component is equal to '6'.

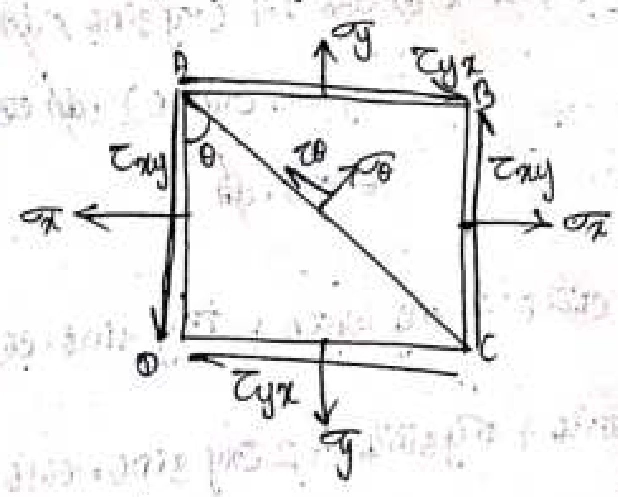
Normal & shear stress along a given plane :-

Plane stress condition (2D stress condition) :-

When stress is acting along two perpendicular direction only then the loading condition of the element is known as Plane stress condition.

Two diamensional stress condition is also known as Plane stress condition.

If σ_x, σ_y and τ_{xy} are acting on the element then the stress condition is known as Plane stress condition and the element is free from loading along 'z' direction.



In plane stress condition along

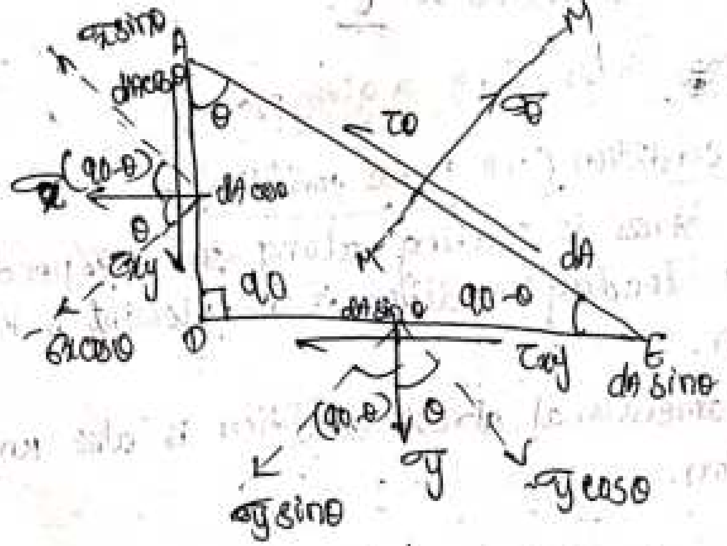
XY Plane $\longrightarrow \sigma_x, \sigma_y, \tau_{xy}$

YZ Plane $\longrightarrow \sigma_y, \sigma_z, \tau_{zy}$

XZ Plane $\longrightarrow \sigma_x, \sigma_z, \tau_{zx}$

10 Mark

$$\boxed{\begin{aligned}\sigma_\theta &= \left(\frac{\sigma_x+\sigma_y}{2}\right)+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta \\ \tau_\theta &= \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta-\tau_{xy}\cos 2\theta\end{aligned}}$$



Net force acting along plane MM = 0

Therefore, $(\sigma_x \cos\theta \times \cancel{dA}\cos\theta) + (\sigma_y \sin\theta \times \cancel{dA}\sin\theta) + \tau_{xy}\cos(90-\theta)\cdot\cancel{dA}\cos\theta + \tau_{xy}\cos\theta\,\cancel{dA}\sin\theta = \sigma_\theta \cdot \cancel{dA}$

$$\Rightarrow \sigma_\theta = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\sin\theta\cdot\cos\theta + \tau_{xy}\cos\theta\cdot\sin\theta$$

$$= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cdot\cos\theta$$

$$= \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy}(2\sin\theta\cdot\cos\theta)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x\cos 2\theta}{2} + \frac{\sigma_y}{2} - \frac{\sigma_y\cos 2\theta}{2} + \tau_{xy}\cdot\sin 2\theta$$

$$\Rightarrow \boxed{\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right)+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta}$$

NOTE:-

θ value is consider from the plane where maximum normal stress is acting.

Net force acting along NN Plane = 0

$\Rightarrow (\tau_\theta\, dA) - (\tau_{xy} \cos\theta \cdot x\, dA \cos\theta) + \tau_{xy} \cos(90-\theta)\, dA \sin\theta + \sigma_x \sin\theta \cdot dA \cos\theta - \sigma_y \sin(90-\theta) \times dA \sin\theta = 0$

$\Rightarrow \tau_\theta \cdot dA = \frac{2}{2}\sigma_y \cdot \cos\theta \cdot dA \sin\theta - \frac{2}{2}\sigma_x \sin\theta \cdot dA \cos\theta + \tau_{xy} \cos\theta \cdot dA \cos\theta - \tau_{xy} \sin\theta \cdot dA \sin\theta$

$\Rightarrow \tau_\theta = 2\cos\theta \sin\theta \left(\frac{\sigma_y}{2} - \frac{\sigma_x}{2}\right) + \tau_{xy} \cos^2\theta - \tau_{xy} \sin^2\theta$

$\Rightarrow = \sin 2\theta \left(\frac{\sigma_y - \sigma_x}{2}\right) + \tau_{xy} (\cos^2\theta - \sin^2\theta)$

$$\Rightarrow \boxed{\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - \tau_{xy} \cos 2\theta}$$

*

If in question is $\sigma_x > \sigma_y$ then direct put angle $= \theta$.

If in question is $\sigma_x < \sigma_y$ then put angle $= 90 - \theta$

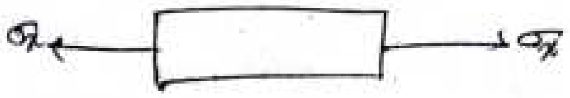
like $\sigma_x = 20$, $\sigma_y = 30$ & $\theta = 30°$ then put $90 - \theta = 60°$

$\sigma_x = 30$, $\sigma_y = 20$ & $\theta = 30°$ then put $\theta = 30$

Case-I

imp **Uniaxial stress condition :-**

Here
$\sigma_y = 0$
$\tau_{xy} = 0$



So, $\sigma_\theta = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta = \frac{\sigma_x}{2}(1+\cos 2\theta)$

$= \sigma_x \cos^2\theta \quad \left[\because \frac{1+\cos 2\theta}{2} = \cos^2\theta\right]$

$\tau_\theta = \frac{\sigma_x}{2}\sin 2\theta$

$\sigma_{\theta\,max} = \sigma_x$, when $\cos^2\theta = 1 \Rightarrow \cos\theta = \pm 1 \Rightarrow \cos\theta = \cos 0^\circ \Rightarrow \boxed{\theta = 0^\circ}$

$\tau_{\theta\,max} = \frac{\sigma_x}{2}$, when $\sin 2\theta = 1 \Rightarrow \sin 2\theta = \sin 90^\circ \Rightarrow 2\theta = 90^\circ$
$\Rightarrow \boxed{\theta = 45^\circ}$

NOTE

Ductile materials are weak in shear but brittle materials are weak in tension.

Hence the failure plane incase of brittle materials will be zero degree and incase of ductile materials the failure plane will be at an angle of 45°.

Case-II

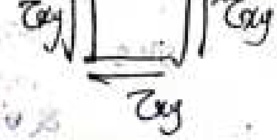
imp **Pure shear condition :-**

Only shear stress is acting on the element.

Here, $\sigma_x = 0$, $\sigma_y = 0$

$\sigma_\theta = \tau_{xy} \sin 2\theta$

$\tau_\theta = -\tau_{xy} \cos 2\theta$



$\sigma_{\theta max} = \tau_{xy}$ if $\sin 2\theta = 1 \Rightarrow \sin 2\theta = \sin 90° \Rightarrow 2\theta = 90° \Rightarrow \boxed{\theta = 45}$

$\tau_{\theta max} = -\tau_{xy}$ if $\cos 2\theta = 1 \Rightarrow \cos 2\theta = \cos 0° \Rightarrow 2\theta = 0 \Rightarrow \boxed{\theta = 0°}$

Hence incase of ductile materials subjected to pure shear the failure will be at an angle 0°. But incase of brittle material subjected to pure shear the failure will be at an angle of 45°.

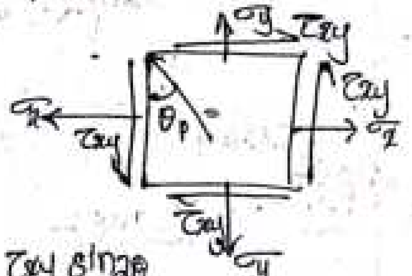
~~Principle~~

Principal stress:—

The maximum or minimum value of normal stress is known as principal stress.

· Maximum Normal stress → Major principal stress

Minimum Normal stress → Minor principal stress



we know

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Here, Normal stress to be maximum or minimum

$$\frac{d\sigma_\theta}{d\theta} = 0$$

$$\Rightarrow \frac{d}{d\theta}\left\{\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta\right\} = 0$$

$$\Rightarrow \frac{d}{d\theta}\left(\frac{\sigma_x + \sigma_y}{2}\right)^{0} + \frac{d}{d\theta}\left\{\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta\right\} + \frac{d}{d\theta}\tau_{xy}\sin 2\theta = 0$$

$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{2}\right)\frac{d\cos 2\theta}{d\theta} + \tau_{xy}\cdot\frac{d\sin 2\theta}{d\theta} = 0$$

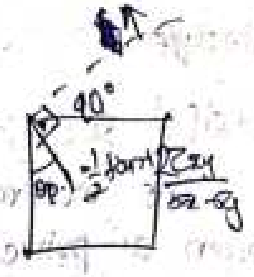
$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{2}\right)(-\sin 2\theta \times 2) + \tau_{xy}(\cos 2\theta \times 2) = 0$$

$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{\cancel{2}}\right)(-\sin 2\theta \times \cancel{2}) = -2\tau_{xy}\cos 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

$$\Rightarrow 2\theta = \tan^{-1}\frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

$$\Rightarrow \boxed{\theta_p = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$$



* So angle between two principal plane is always 90°.

we know

$$\tan\{2(90+\theta_p)\} = \tan(180+2\theta_p) = \tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

~~Define~~

2 Marks

Principal plane:-

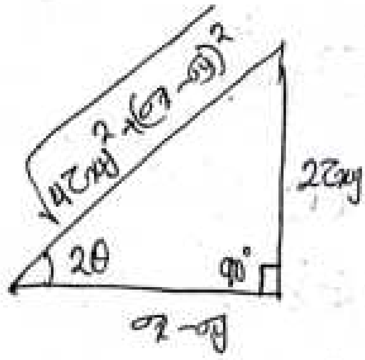
The plane ~~along which~~ perpendicular to which normal stress is either maximum or minimum is known as principal plane.

~~Defi~~ The stress acting on principal plane is known as principal stress.

2 Marks

Q = Define principal stress and principal plane.

$$\tan 2\theta_p = \frac{2\tau xy}{(\sigma x - \sigma y)}$$



Here

$$\sin 2\theta = \frac{p}{h} = \frac{2\tau xy}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}}$$

$$\cos 2\theta = \frac{b}{h} = \frac{\sigma x - \sigma y}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}}$$

we know,

$$\sigma\theta = \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right)\cos 2\theta + \tau xy \sin 2\theta$$

$$= \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right)\frac{(\sigma x - \sigma y)}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}} + \tau xy \cdot \frac{2\tau xy}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}}$$

$$= \left(\frac{\sigma x + \sigma y}{2}\right) + \frac{(\sigma x - \sigma y)^2}{2\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}} + \frac{2\tau xy^2}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}}$$

$$= \left(\frac{\sigma x + \sigma y}{2}\right) + \frac{1}{\sqrt{(\sigma x - \sigma y)^2 + 4\tau xy^2}}\left\{\frac{(\sigma x - \sigma y)^2}{2} + 2\tau xy^2\right\}$$

$$\cancel{= \left(\frac{\sigma x + \sigma y}{2}\right) + \frac{1}{2\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2} + \tau xy^2 \times 2\left\{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2\right\}}$$

$$\cancel{\Rightarrow \sigma_{1,2} = \frac{\sigma x + \sigma y}{2}}$$

$$= \left(\frac{\sigma x + \sigma y}{2}\right) + \frac{1}{\sqrt{4\left\{\frac{(\sigma x - \sigma y)^2}{4} + \tau xy^2\right\}}} \times \left\{\frac{(\sigma x - \sigma y)^2}{2} + 2\tau xy^2\right\}$$

$$= \left(\frac{\sigma x + \sigma y}{2}\right) \frac{1}{2\sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}} \times 2\left\{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2\right\}$$

$$\Rightarrow \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{\cancel{2}}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \times \cancel{2}$$

$$\Rightarrow \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 \rightarrow$ Major principal stress (+)

$\sigma_2 \rightarrow$ Minor principal stress (−)

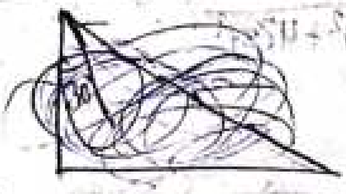
Q) An element in a strain ~~by~~ body is subjected to tensile stress of 200 MPa in horizontal direction and 150 MPa in vertical direction, each of the above stress is accompaned by a shear stress of 30 MPa such that, when associated with the major tensile stress tends to rotate the element in clockwise direction find

i) Magnitude of normal and shear stress on a section inclined at an angle of 30° with the major tensile stress

ii) Principal stress and their direction

iii) Maximum shear stress and it's direction

Soln



Here $\sigma_x = 200$ MPa

$\sigma_y = 150$ MPa

$\tau_{xy} = 30$ MPa

$\theta = 30°$

i) we know

$$\sigma_\theta = \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right) + \cos 2\theta + \tau xy \sin 2\theta$$

$$= \left(\frac{200 + 150}{2}\right) + \left(\frac{200 - 150}{2}\right) + \cos 60 + 30 \times \sin 60$$

$$= 175 + \left(25 \times \frac{1}{2}\right) + \left(30 \times \frac{\sqrt{3}}{2}\right)$$

$$= 213.48 \text{ MPa}$$

$$\tau_\theta = \left(\frac{\sigma x - \sigma y}{2}\right) \sin 2\theta - \tau xy \cos 2\theta$$

$$= \left(\frac{200 - 150}{2}\right) \sin 60 - 30 \times \cos 60$$

$$= 25 \times \frac{\sqrt{3}}{2} - 30 \times \frac{1}{2}$$

$$= 6.65 \text{ MPa}$$

ii) Principal stresses :-

Major principal stress $(\sigma_1) = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}$

$$= \left(\frac{200 + 150}{2}\right) + \sqrt{\left(\frac{200 - 150}{2}\right)^2 + 30^2}$$

$$= 175 + \sqrt{25^2 + 30^2}$$

$$= 175 + \sqrt{625 + 900}$$

$$= 175 + \sqrt{1525}$$

$$= 175 + 39.05$$

$$= 214.05 \text{ MPa}$$

Minor principal stress $(\sigma_2) = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 175 - \sqrt{25^2+30^2}$

$= 175 - 39.05$

$= 135.9$ MPa

(iii)

Direction

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$= \frac{2 \times 30}{200-150}$

$= \frac{60}{50}$

$\Rightarrow \tan 2\theta_p = 1.2$

$\Rightarrow 2\theta_p = \tan^{-1}(1.2)$

$\Rightarrow \theta_p = \frac{1}{2}\tan^{-1}(1.2)$

$\Rightarrow \theta_p = 25.09°$

direction of another principal plane $= 90° + 25.09$

$= 115.09$

(iii) Maximum shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{214.05 - 135.94}{2}$$

$$= \frac{78.11}{2}$$

$$= 39.055 \text{ MPa}$$

direction

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$= \frac{-(200-150)}{2 \times 30}$$

$$= \frac{-50}{60}$$

$$\Rightarrow 2\theta_s = \tan^{-1}\left(\frac{-5}{6}\right)$$

$$\Rightarrow 2\theta_s = 39.8^\circ$$

$$\Rightarrow \theta_s = \frac{39.8}{2}$$

$$\Rightarrow \theta_s = 19.9^\circ$$

Principal Shear Plane :-

The plane along which shear stress is either maximum or minimum is known as Principal shear plane.

$$\cancel{\frac{d\tau_\theta}{d\theta}}$$

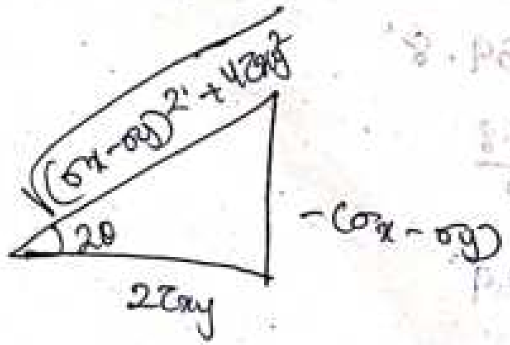
$$\frac{d\tau_\theta}{d\theta} = 0$$

$$\Rightarrow \frac{d}{d\theta}\left\{\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta\right\} = 0$$

$$\Rightarrow \frac{\sigma_x - \sigma_y}{2} \times 2\cos 2\theta - \tau_{xy}(-2\sin 2\theta) = 0$$

$$\Rightarrow \frac{\sigma_x - \sigma_y}{\cancel{2}} \times \cancel{2}\cos 2\theta = \tau_{xy}(-2\sin 2\theta) = 0$$

$$\Rightarrow \boxed{\tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}}$$



$$\sin 2\theta = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}, \quad \cos 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{\cancel{2}} \times \frac{\cancel{2}\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} + \tau_{xy}\frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\Rightarrow \boxed{\sigma_\theta = \frac{\sigma_x + \sigma_y}{2}}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$\tan 2\theta_p \times \tan 2\theta_s = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} \times \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$= -1$$

$$\Rightarrow 2\theta_p \perp 2\theta_s \qquad \left[\begin{array}{c} m_1 m_2 = -1 \\ \text{(slop)} \end{array}\right]$$

__Maximum shear stress:—__

$$\tau_{\theta max} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{\sigma_x - \sigma_y}{2} \times \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} - \tau_{xy} \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$= \frac{-\{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\}}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\Rightarrow \boxed{\tau_{\theta max} = \frac{\sigma_1 - \sigma_2}{2}}$$

Note

(1) $\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

(2) $\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

(3) $\sigma_{1 or 2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

(4) $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$ → direction

(5) Shear stress acting on the principal plane is always 0.

(6) $\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$

(7) Value of Normal stress on principal shear plane $= \frac{\sigma_x + \sigma_y}{2}$

(8) Angle between two principal plane is always 90°.

(9) Angle between principal plane and principal shear plane will be 45°.

(10) Maximum shear stress $= \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$ → Radius of Mohr's circle represent maximum shear stress

(11) Resultant stress $\sigma_R = \sqrt{\sigma_\theta^2 + \tau_\theta^2}$

(12) Direction of resultant $(\phi) = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right)$

Graphical Method :—

Mohr's circle method :—

Case-I → Two like and unequal stress

(i) Take horizontal axis as σ-axis and the vertical axis as τ axis.

(ii) Take 'O' as origin and $OD = \sigma_x$, $OA = \sigma_y$ along σ axis

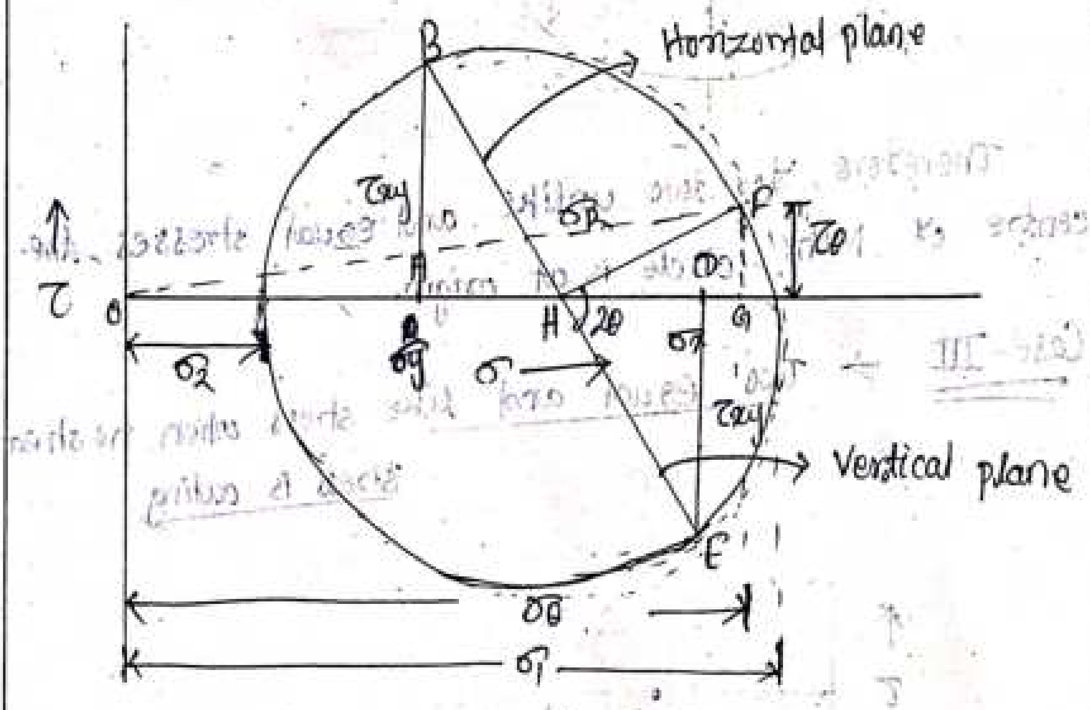
(iii) Take $AB = DE = \tau_{xy}$ along τ-axis

(iv) Join BE which will intersect the σ-axis at point 'H' and draw a circle taking BH or HE as radius.

(v) Take HE as base and draw a angle $\angle FHE = 2\theta$

(vi) Draw a perpendicular from F towards σ_x is which is FG

(vii) The distance OG and FG represents σ_θ and τ_θ respectively.



Note

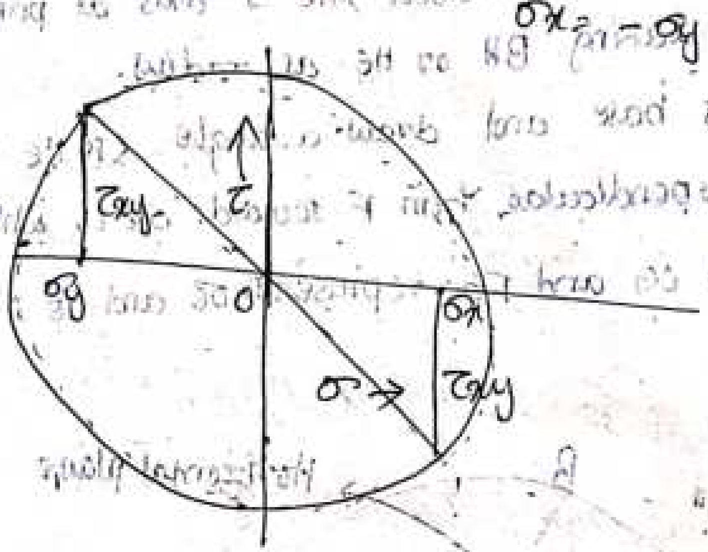
(i) Radius of the Mohr's circle represents a plane.

(ii) Angle is double incase of Mohr's circle.

(iii) Radius of Mohr's circle represents maximum shear stress.

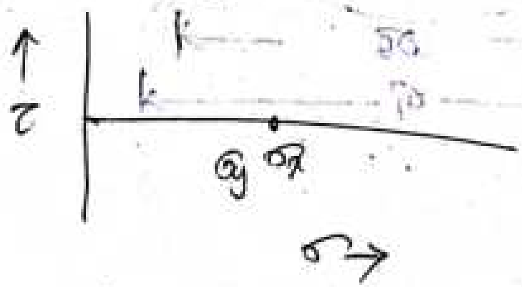
(iv) Maximum shear stress $= \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2} = \frac{\sigma_1 - \sigma_2}{2}$

Case-II → Two unlike and equal stress



Therefore, for two unlike and equal stresses, the centre of Mohr circle is at origin.

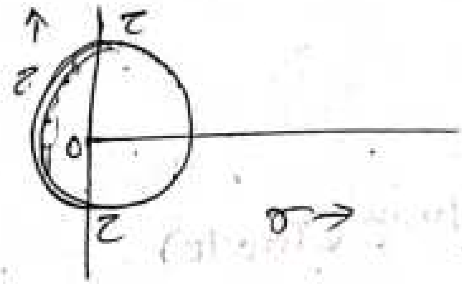
Case-III → Two equal and like stress when no shear stress is acting



Mohr's circle reduces to a point when two equal and like stresses act with no shear stress.

Case - IV → Pure shear

$\sigma_x = 0,\ \sigma_y = 0,\ \tau_{xy} = \tau$



In case of pure shear, centre of Mohr's circle is at origin and radius of Mohr's circle is equal to 'τ'.

Q) The principal stress at a point in a bar are 100 N/mm² (tensile) and 50 N/mm² (compressive). Determine the resultant stress in magnitude and direction, if the plane is inclined at 60° to the axis of major principal stresses. Draw the diagram showing resultant stress & direction.

Soln

$\sigma_x = 100\ N/mm^2$

$\sigma_y = -50\ N/mm^2$

$\sigma_R = ?$

$\theta = 60°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \cancel{\tau_{xy}\sin 2\theta}$$

$$= \frac{100-50}{2} + \frac{100-(-50)}{2}\cos 120$$

$= 25 + 75 \times \left(-\frac{1}{2}\right)$

$= 25 - 37.5$

$= -12.5\ N/mm^2$ (compressive)

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{100 - (-50)}{2} \sin 120$$

$$= 75 \times \frac{\sqrt{3}}{2}$$

$$= 64.95 \text{ N/mm}^2 \text{ (Tensile)}$$

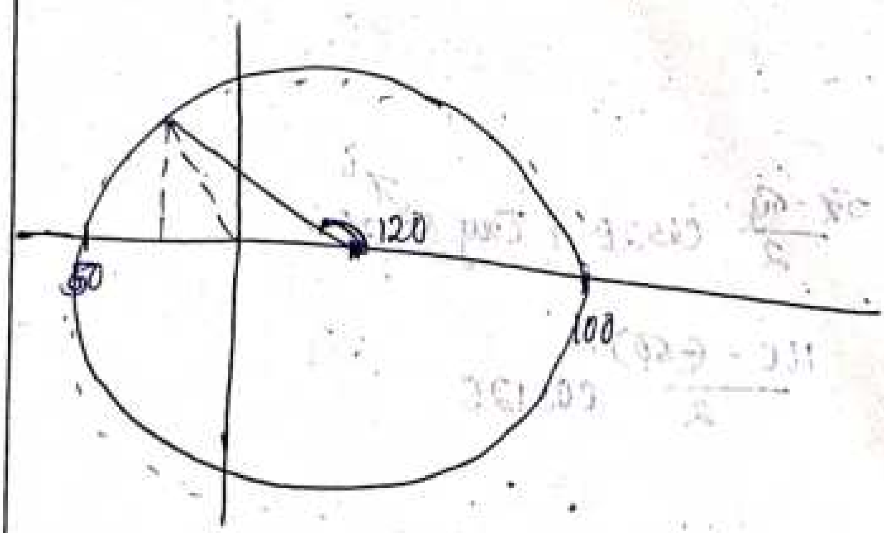
$$\sigma_R = \sqrt{\sigma_\theta^2 + \tau_\theta^2}$$

$$= \sqrt{(-12.5)^2 + (64.5)^2}$$

$$= 65.70 \text{ N/mm}^2$$

$$\boxed{\text{Direction of resultant } (\phi) = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right)}$$

$$\text{Direction} = \tan^{-1}\left(\frac{64.95}{-12.5}\right) = 79.10^\circ$$



Q) A steel plate subjected to tensile stress of 200 MPa and 160 MPa at right angle to each other. Determine the normal & shear stress in a plane inclined at 30° with the 200 MPa stress plane.

Soln

~~Normal stress (σ_θ) =~~

$\sigma_x = 200$ MPa

$\sigma_y = 160$ MPa

$\tau_{xy} = 0$

$\theta = 30°$

Normal stress (σ_θ) = $\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= \frac{200+160}{2} + \frac{200-160}{2}\cos 60° + 0 \times \sin 60$

$= 180 + 20 \times \cos 60$

$= 180 + \cancel{20}^{10} \times \frac{1}{\cancel{2}}$

$= 190$ MPa

Shear stress (τ_θ) = $\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$= \frac{200-160}{2}\sin 60 - 0 \times \cos 60$

$= 20 \times \frac{\sqrt{3}}{2}$

$= 17.32$ MPa.

Q) The principal stresses at a point across two perpendicular planes are 75 MN/m2 (tensile) and 35 MN/m2 (tensile). Find the normal, tensetial stress (τ_θ) and the resultant stress and obliguaty angle (ϕ) on a plate at 20° with minor principal plane.

Soln

$$\sigma_x = 75 \text{ MN/m}^2$$

$$\sigma_y = 35 \text{ MN/m}^2$$

$$\theta = 90-20 = 70$$

Normal stress (σ_θ) $= \dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$$= \frac{75+35}{2} + \frac{75-35}{2}\cos 140 + 0 \times \sin 2\theta$$

$$= 55 + 20 \times (-0.76)$$

$$= 55 + (-15.2)$$

$$= 55 - 15.2$$

$$= 39.8 \text{ MN/m}^2$$

Shear stress (τ_θ) $= \dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$$= \frac{75-35}{2}\sin 140 - 0 \times \cos 140$$

$$= 20 \times (0.64)$$

$$= 12.8 \text{ MN/m}^2$$

$$\text{Resultant} = \sqrt{\sigma_\theta^2 + \tau_\theta^2} \qquad \phi = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right)$$

$$= \sqrt{(39.8)^2 + (12.8)^2} \qquad = \tan^{-1}\left(\frac{12.8}{39.8}\right)$$

$$= \sqrt{1584.04 + 163.84} \qquad = 17.82$$

$$= \sqrt{\cancel{1547.88}} = \sqrt{1711.88}$$

$$= \cancel{37.07\ MN/m^2}$$

$$= 41.37\ MN/m^2$$

Q) Determine the major principal stress and minor principal stress at a point of a machine component, if normal stress are 200 MPa and 100 MPa respectively and the shear stress component = 40 MPa.

Soln

$\sigma_x = 200$ MPa

$\sigma_y = 100$ MPa

$\tau_{xy} = 40$ MPa

$$\text{Major principal stress } (\sigma_1) = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{200 + 100}{2} + \sqrt{\left(\frac{200 - 100}{2}\right)^2 + 40^2}$$

$$= 150 + \sqrt{50^2 + 40^2}$$

$$= 150 + \sqrt{2500 + 1600}$$

$$= 150 + 64.03$$

$$= 214.03\ \text{MPa}$$

Minor principal stress(σ_2) $= \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{200+100}{2} - \sqrt{\left(\frac{200-100}{2}\right)^2 + 40^2}$

$= 150 - \sqrt{50^2 + 40^2}$

$= 150 - \sqrt{2500+1600}$

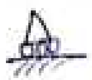
$= 150 - 64.03$

$= 85.97$ MPa

SFD & BMD

Types of supports:-

No of reactions forces

1) fixed –  → 3
2) Roller –  → 1
3) Hinged –  → 2

Types of load:-

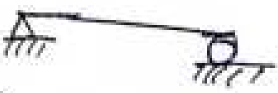
Beam:-

Beam is a horizontal member which is design to carry vertical load & moments acting on it and safty transfor it to the adjucent columns.

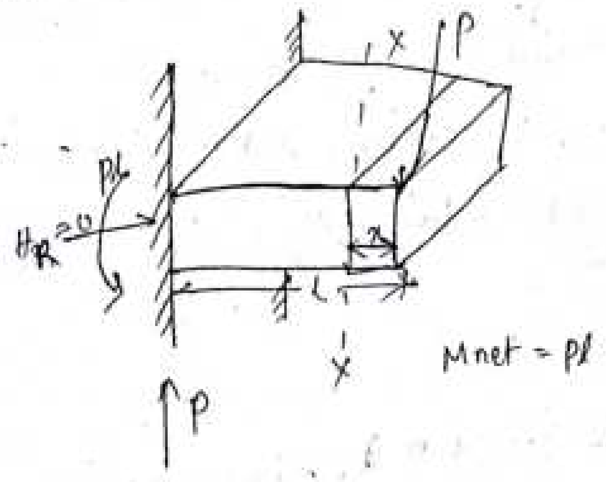
or

Beam is a member which bends.

Types of beam:-

1) fixed beam 
2) simply supported beam 
3) Cantilever beam 
4) Propped cantilever beam → 
5) continuous beam → resting on More than 2 supports 
6) over hanging " → 

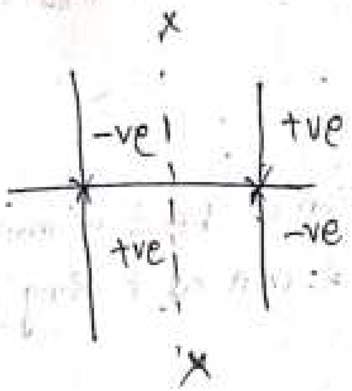
Shear force :-



$$\boxed{SF)_{xx} = P}$$

⇒ Shear force at a section may be defined as the net/summation of all force including reactions acting at that section either to the left or to the right of the section.

Sign convention for shear force :-



RUN → Right hand side upword force negative.

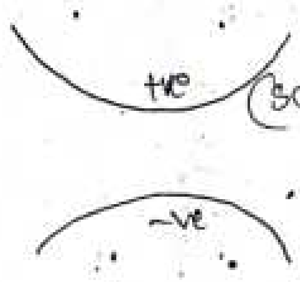
Bending moment :-

Bending moment at a section may be defined as the summation of all the moments either to the left or to the right of the section.

$$\boxed{M_{net} = -Px}$$

Sign convertion for bending moment :-

All ~~downward force~~ moments developed due to downword forces will be taken as negative. and moments developed due to upword forces is taken as positive.



(Sagging bending moment)

(Hogging bending moment)

Shear force diagram:-

The graphical representation of the shear force across the length of the beam is known as shear force diagram

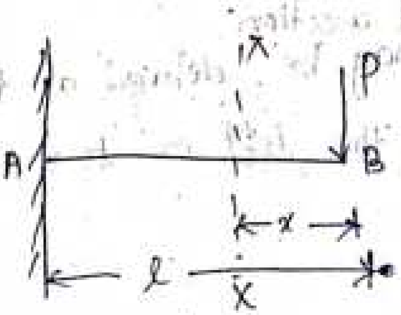
Bending moment diagram:-

The graphical representation of bending moment across the Length of the beam is known as bending moment diagram.

Case-1

Cantilever with concentrated load at free end
(concentrated → Point load)

~~Let us~~



Let us consider a section 'xx' at a distance of 'x' from the free end of the beam.

Considering right hand portion of section 'xx'

$$SF)_{xx} = +P$$

$$BM)_{xx} = -Px$$

So, shear force at $x=0$, shear force at B

$$SF)_{xx} = +P$$

shear force at $x=l$, shear force at A

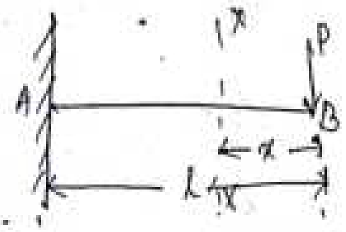
$$SF)_{xx} = +P$$

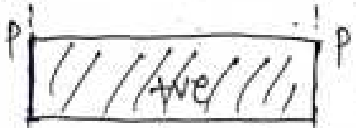
• Bending moment at $x=0$, bending moment at B

$$BM)_{xx} = 0$$

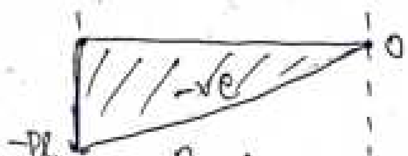
BM at $x=l$, BM at A

$$BM)_{xx} = -Pl$$





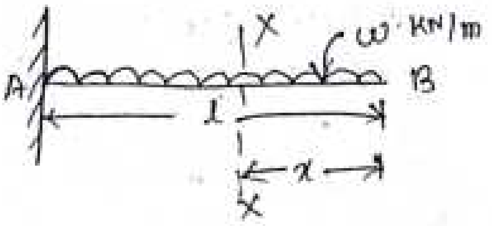
Shear force diagram



Bending moment diagram

Case- 2

Cantilever with uniformly distributed load :-



Let us consider a section 'XX' at a distance of 'x' from the free end of the beam.

Considering right hand portion of section 'XX'.

$SF)_{xx} = +wx$

$BM)_{xx} = -wx \times \frac{x}{2}$

So, SF at $x=0$, S.F at B

$SF)_{xx} = 0$

SF at $x=l$, S.F at A

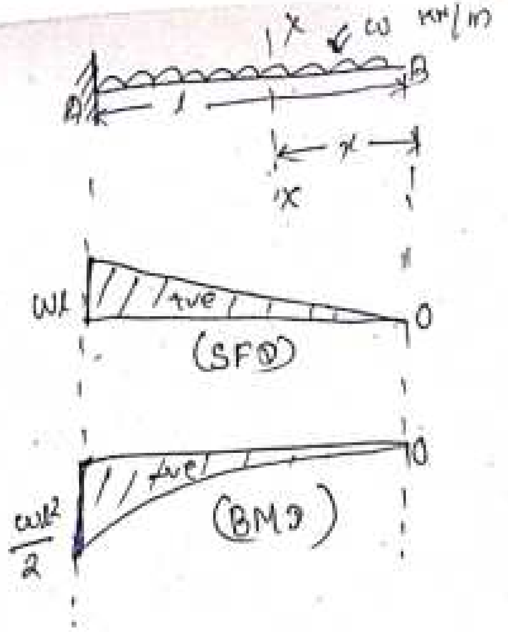
$SF)_{xx} = wl$

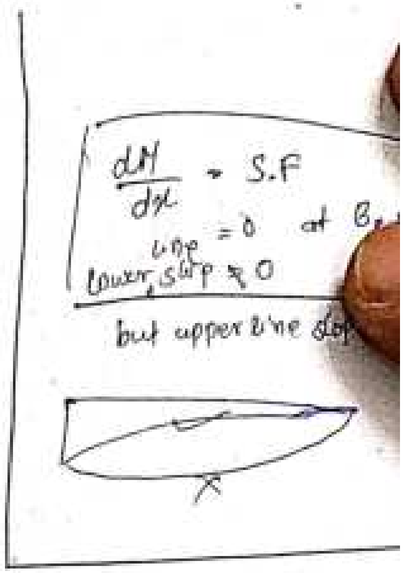
BM at $x=0$, BM at B

$BM)_{xx} = 0$

BM at $x=l$, BM at A

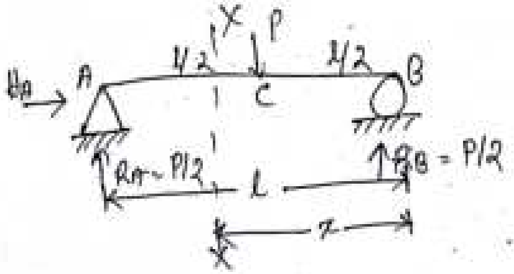
$BM)_{xx} = -wl \times \frac{l}{2} = -\frac{wl^2}{2}$





Case-3

Simply supported beam with point load at centre :-



Let us consider a section 'xx' at a distance of 'x' from support 'B'

$\Sigma H = 0 \Rightarrow P - R_A - R_B = 0 \Rightarrow R_A + R_B = P$

$\Sigma V = 0 \Rightarrow (R_B \times l) - (P \times \frac{l}{2}) = 0$

$\Rightarrow R_B = P/2$

$\Rightarrow R_A = P - P/2 = P/2$

$S.F)_{xx} = \frac{-P}{2} + P = \frac{P}{2}$

$B.M)_{xx} = +(\frac{P}{2} \times x) - P \times (x - \frac{l}{2})$

So, S.F at $x=0$, at B

$S.F_B = P/2$

S.F at $x=l$, at A

$S.F_A = P/2$

S.F at $x=l/2$ at C

$S.F_C = P/2$

B.M at $x=0$, at B

$B.M_B =$

BM at $x=l$, at A

$B.M_A = 0$

BM at $x=\frac{l}{2}$, at C

$$B.M = \left(\frac{P}{2}\times\frac{l}{2}\right) - P\times\left(\frac{l}{2}-\frac{l}{2}\right)$$

$$= \frac{Pl}{4}.$$

AC

$$S.F)_{xx} = -\frac{P}{2} + P = P/2$$

$$B.M)_{xx} = \frac{P}{2}x - P\left(x-\frac{l}{2}\right)$$

CB

$$S.F)_{xx} = \frac{-P}{2}$$

$$B.M)_{xx} = +\frac{P}{2}\times x$$

S.F. at $x = l$, $SF_A = P/2$

at $x = l/2$, $SF_C = P/2$

at $x = l/2$, $SF_C = -P/2$

B.M at $x = l$, $BM_A = 0$

at $x = l/2$, $BM_C = \frac{P}{2} \cdot \frac{l}{2} - P\left(\frac{l}{2} - \frac{l}{2}\right)$

$= \frac{Pl}{4}$

at $x = l/2$, $BM_C = \frac{Pl}{4}$

at $x = 0$, $BM_B = 0$